# NOMOGRAPHIC CHARTS

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C. ALBERT KULMANN

First Edition

McGRAW-HILL BOOK COMPANY, INC.
New York Toronto London
1951

NOMOGRAPHIC CHARTS

To

the person who prodded me into finally completing the job

# PREFACE

This volume is a collection of charts found by the author to effect enormous savings in time in the routine work that has gone through his office over a long period of years. Some of the charts are redrawn from originals that have been in use for over 20 years, and many of them have appeared in the technical press.

"Nomographic Charts" is not intended to be an engineering handbook, though it might well be expanded to approach that designation. Neither is it intended as a textbook on graphical representation, nor is it offered as giving the best possible graphical method for solution of the problems covered.

It should be borne in mind that any charts such as these cannot be taken as giving exact solutions for the problems covered. Rather, they should be taken as something between ordinary slide-rule computation and exact numerical computation. For one thing, the charts were hand-drawn, not machine-scaled, and while the originals were scaled by use of a glass, in a size much larger than the page reproductions, there is a certain amount of unavoidable human error. Then, too, in the photoreproduction to reduce size and in the process of making plates, there may be some distortion in the lens, though this is of small magnitude and applies only to the purely alignment nomographs and the alignment sections of those of composite type.

On the other hand, in most cases the accuracy is far beyond that of slide-rule computation, despite the machine scaling of the latter. The basis for this statement is that in many problems the values of certain of the variables are confined to very narrow limits, perhaps only a small fraction of the slide-rule scale. By expansion of such small sections of what would be the normal slide-rule scale, the accuracy of that portion of the problem is greatly increased, and this increase in most cases far outweighs the small inaccuracies inherent in this collection.

Certainly, then, these charts present solutions of an accuracy which does not suffer by comparison with slide-rule computation. But by far the greatest value of these charts is in the timesaving effected by obviating the need for looking up in various tables the values of the various items that enter into a problem. For instance, where the factors that enter into a problem may be modified by exponential functions, or varying empirical multipliers, or similar variations, the chart scalings have taken them into account and reference to tables or auxiliary curve sheets is unnecessary. This point is of particular value where the factors may involve odd exponential functions or empirical factors based upon the relationship between two or more of the basic factors.

The choice of design for the individual charts deserves a word of explanation. The alignment type of nomograph has been a particular hobby

of the author since school days, but in many of the following charts, intersection nomographs or combinations of intersection and alignment nomographs have been the final choice. For most of the problems for which these types of solutions were selected, there have appeared in the technical press nomographs of the purely alignment type, based, however, upon approximate rather than exact mathematical solutions. Examples of such are found in the charts for steam quality determination by steam calorimeter, for the length of an open belt passing around two pulleys, for the magnitude of surges in pipe lines due to water hammer, and others. In each case, the curved lines in the intersection type of nomograph may not depart far from straight lines, but they do depart. And the charts of purely alignment type that purport to solve such problems are based upon straight-line relationships which approach the curve, but which yet remain approximations. True, the chart based upon the exact solution may take far more time in preparation, but once made, it requires no more time for solution than another chart, and it brings an answer based upon mathematical truth.

One more item might well be touched upon in discussing the general use of these charts, and that has to do, again, with accuracy. This has to do with the accuracy of *any* engineering computation, and, certainly, the accuracy of the final answer to any problem can be no more accurate than the basic data upon which the computation is based. And so, since it is impossible to predict exactly the modulus of elasticity of a certain piece of steel, or the friction factor of a pipe line, or the average temperature and resultant resistance of a copper conductor, or similar items that enter into every engineering problem, it is equally impossible to obtain any truly exact answers. Thus, if the accuracy of the result is commensurate with the accuracy of the basic data, there is no more that can be expected. And it is believed that the accompanying charts do fulfill this requirement.

C. Albert Kulmann

San Francisco, Calif.
*June,* 1951

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# Group I

# FUNCTION SCALES

This group of charts has to do, for the most part, with functions of numbers. Some of the charts, it is true, present data easily available in tabular form in handbooks, but these were included for two reasons. First, it had been found very convenient to have these in the same form and in the same group with the others. But, second, and more important, visual interpolation between scale points is easier and faster than interpolation between the tabulated figures.

There are, however, a number of exponential functions of numbers, used more or less widely in engineering, for which tables are limited in scope or nonexistent. Some of these appear here.

Naturally, in the absence of applicable tables of such exponential functions, computation becomes an involved, tedious task—looking up a logarithm, multiplying by the exponent, and then going back for an antilogarithm, with the problem of keeping the decimal point in the right place. But with these charts, the problem becomes even easier than a single reference to a table, in most cases. For any necessary interpolation is visual, involving no determination of fractional difference in the given data, to be applied then in determining a differential in the result—it is plain to the eye.

Certain of these exponential functions have only limited application, and their variation is between rather fixed limits, and for them, with these charts, there is no worry as to the decimal point. In others, where there may be no such limits in the magnitude of the variable, the placing of the decimal point is a simple matter by reason of the method used in scaling. A study of a chart will make procedure obvious and easy. However, in each case, the use of the particular chart is explained, together with proper treatment of values outside the main scaling.

In the exponential-function charts, there appears, in each, a primary scaling, which covers a basic range. Then at certain index points on the scales appear values beyond the primary range. For instance, on the first chart of this group, *Fifth Power and Root,* the $N$ scale runs from 1.0 to 10, with the $N^5$ scale from 1.0 to 100,000. At the ends of the scales, however, appear auxiliary values, with minimum values of 0.1 and 10 and maxima of 1.0 and 100 for $N$, and with corresponding $N^5$ values, 0.00001 and 100,000 minima and 1.0 and 10,000,000,000 maxima. These auxiliary scale values then set the pattern for values beyond the primary scaling, and this is true of many of the other charts.

In these exponential-function charts, recourse is had to the basic theorem that the value of any quantity raised to any exponential power is equal to a quantity that is the product of multiplication of the

same exponential power of all of the factors which produce the original quantity. Thus, if $Y = a \times b$ ($a$ times $b$), then $Y^n = a^n \times b^n$. Accordingly, for values outside the limits of the primary scalings, those values are broken down into their factors. This sounds, in abstract terminology, rather complicated, but in fact it is not.

For instance, still taking the fifth power and root chart for an example, and wanting the fifth power of 18, it is necessary to follow this principle, but one of the factors is taken as 10 and the other as 1.8, both of which do appear, the former on the auxiliary scaling and the other on the primary. Thus, the fifth power of 18 is 100,000 ($10^5$) times 18.9 ($1.8^5$), and the result is 1,890,000, within a fraction of a per cent of the true computed value. Thus, what in abstract description appears complex is really quite simple, and with fractional exponents treated in the same way, the roots of numbers beyond the range of the primary scaling become equally simple.

## Charts in Group I

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# 1. Fifth Power and Root

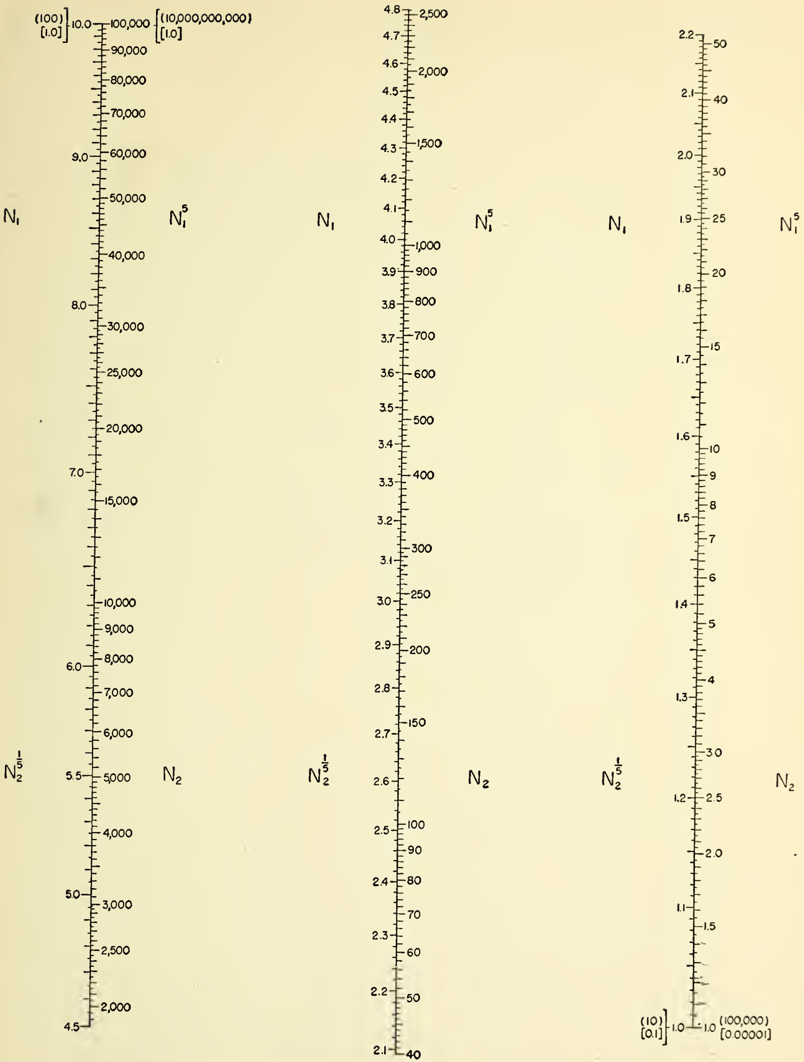
**Method for the determination** of those functions. Tables for these functions are not readily available, and even when they are, a computation of proportional differences, for interpolation, will be necessary more often than not. Where tables are not available, solution becomes a series of computations involving determinations of logarithms and antilogarithms.

The chart, however, combines an accuracy at least equal to tables when subject to interpolation, with quicker and easier interpolation, and a range of coverage beyond any tables known to the author.

**Procedure.** To determine the fifth power of a number within the primary scaling range, that number scaling is located on the $N_1$ scale, interpolating as necessary. Thus, determining $4^5$, the value opposite 4 on the $N_1$ scale appears on the $N_1{}^5$ scale as 1,025 (true value 1,024). Conversely, to determine a fifth root, the given number is located on the $N_2$ scale, and the root is read on the $N^{1/5}$ scale. Thus, solving $3{,}125^{1/5}$, the value opposite 3,125 (interpolating), on the $N_2$ scale, is 5.0 on the $N_2{}^{1/5}$ scale.

Values beyond the primary scalings are broken down into factors, so, for instance, the value of $18^5$ is equal to $10^5 \times 1.8^5$. The fifth powers of the factors are read as 100,000 and 18.9, respectively, so $18^5 = 100{,}000 \times 18.9 = 1{,}890{,}000$. Conversely, $0.7^{1/5} = 0.1^{1/5} \times 7^{1/5} = 0.00001 \times 1.474 = 0.00001474$.

The same principle applies even when three or more factors are required to break down the figures to the limits of the chart.


N1
N1^5
N2^1/5
N2
(100) [1.0] 10.0
100,000 [(10,000,000,000) [1.0]
90,000
80,000
70,000
9.0
60,000
50,000
40,000
8.0
30,000
25,000
20,000
7.0
15,000
10,000
9,000
6.0
8,000
7,000
6,000
5.5
5,000
4,000
5.0
3,000
2,500
2,000
4.5
4.8
2,500
4.7
4.6
2,000
4.5
4.4
4.3
1,500
4.2
4.1
4.0
1,000
3.9
900
3.8
800
3.7
700
3.6
600
3.5
500
3.4
3.3
400
3.2
3.1
300
3.0
250
2.9
200
2.8
2.7
150
2.6
2.5
100
90
2.4
80
70
2.3
60
2.2
50
2.1
40
2.2
50
2.1
40
2.0
30
1.9
25
20
1.8
15
1.7
1.6
10
9
8
1.5
7
6
1.4
5
4
1.3
3.0
1.2
2.5
2.0
1.1
1.5
(10) [0.1] 1.0
1.0 (100,000) [0.00001]

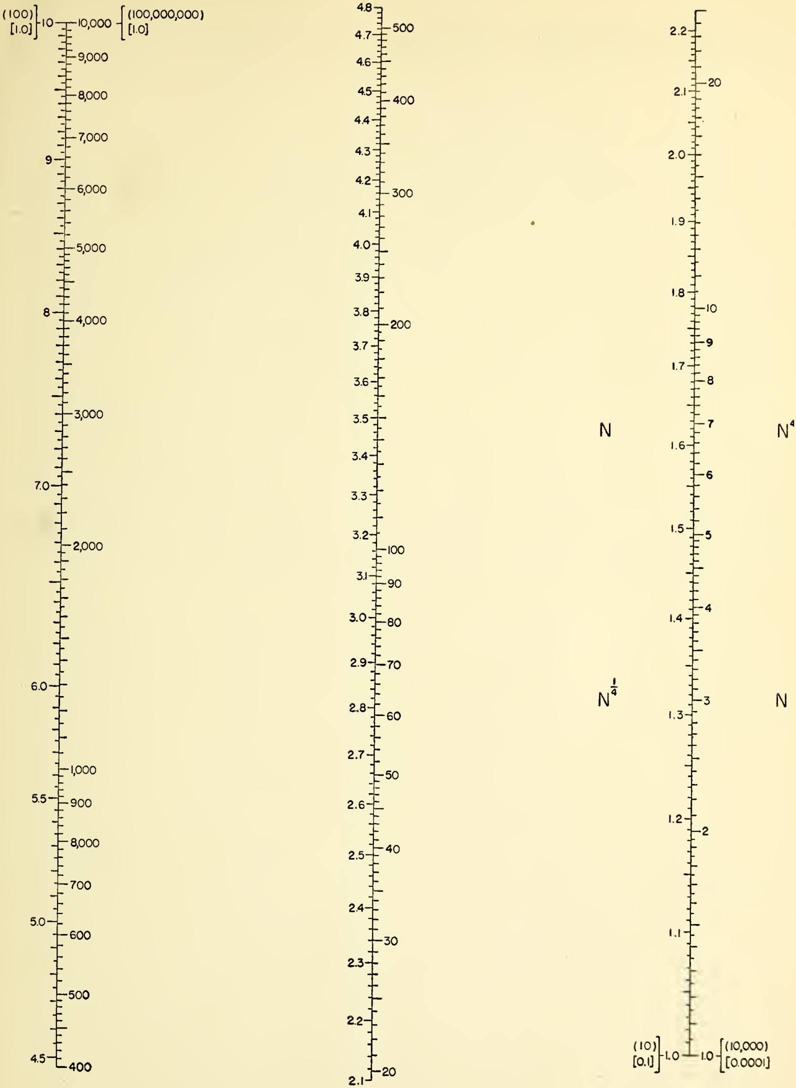
# 2. Fourth Power and Root

**Method for the determination** of those functions. Tables for these functions are not readily available, and even when they are, a computation of proportional differences, for interpolation, will be necessary more often than not. Where tables are not available, the choice is between the double use of square tables with double interpolations, or a series of computations involving determinations of logarithms and antilogarithms.

**Procedure.** To determine the fourth power of a number within the primary scaling range, that number scaling is located on the $N_1$ scale, interpolating as necessary. Thus, determining $5^4$, the value opposite 5 on the $N_1$ scale appears on the $N_1{}^4$ scale as about 623 (true value 625). Conversely, to determine a fourth root, the given number is located on the $N_2$ scale, and the root is read on the $N_2{}^{1/4}$ scale. Thus, solving $420^{1/4}$, the value opposite 420 on the $N_2$ scale is 4.525 on the $N_2{}^{1/5}$ scale.

Values beyond the primary scalings are broken down into factors, so, for instance, the value of $230{,}000^{1/4}$ is equal to $10{,}000^{1/4} \times 23^{1/4}$. The fourth roots of the factors are read as 10.0 and 2.19+, respectively, so $230{,}000^{1/4} = 10.0 \times 2.19+ = 21.9+$. Conversely, $70^4 = 10^4 \times 7^4 = 10{,}000 \times 2{,}395 = 23{,}950{,}000$.

The same principle applies even when three or more factors are required to break down the figures to the limits of the chart.


(100)
[1.0]
10
10,000
(100,000,000)
[1.0]
9,000
8,000
7,000
9
6,000
5,000
8
4,000
3,000
7.0
2,000
6.0
1,000
5.5
900
8,000
700
5.0
600
500
4.5
400
4.8
4.7
500
4.6
4.5
400
4.4
4.3
4.2
300
4.1
4.0
3.9
3.8
200
3.7
3.6
3.5
3.4
3.3
3.2
100
3.1
90
3.0
80
2.9
70
2.8
60
2.7
50
2.6
2.5
40
2.4
30
2.3
2.2
2.1
20
N
N^4
N^(1/4)
N
2.2
2.1
20
2.0
1.9
1.8
10
9
1.7
8
7
1.6
6
1.5
5
4
1.4
3
1.3
1.2
2
1.1
(10)
[0.1]
1.0
1.0
(10,000)
[0.0001]

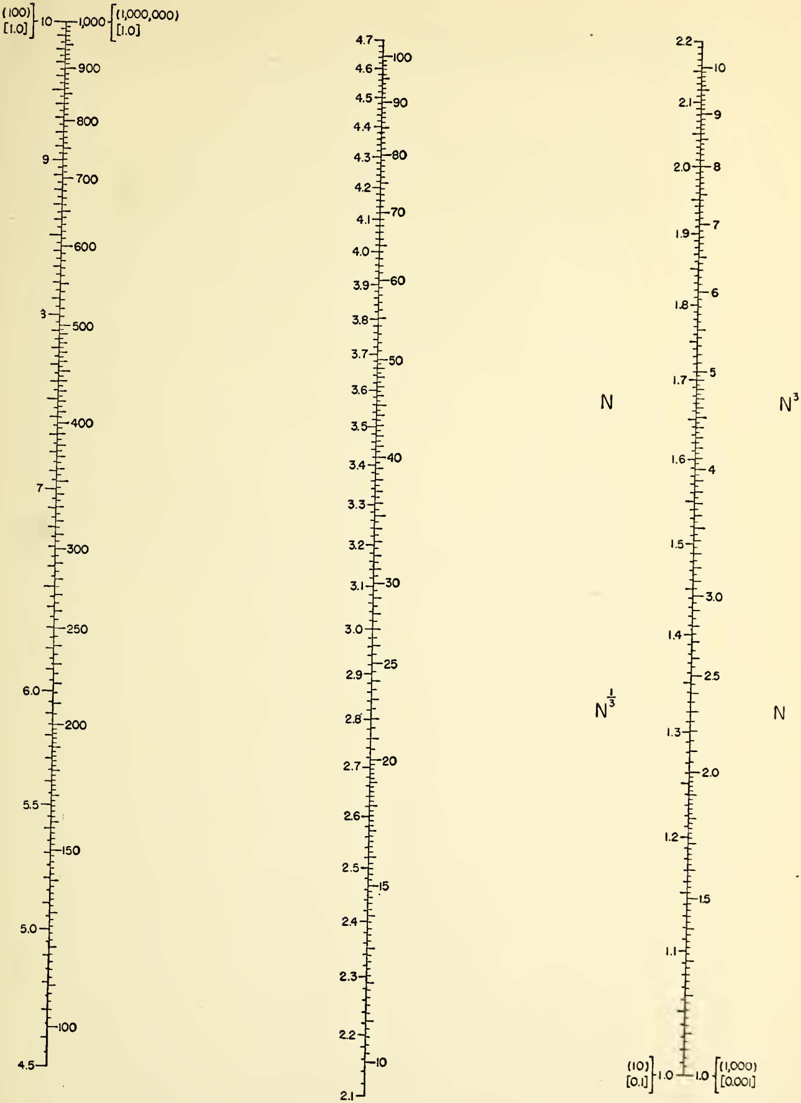
# 3. Cube Power and Root

**Method for the determination** of those functions. Tables for these functions are more or less readily available, it is true, but even so, a computation of proportional differences, for interpolation, is necessary more often than not. Graphical interpolation is far simpler and quicker than numerical; for that reason, as well as to bring such functional determinations within one set of covers, the chart is included.

**Procedure.** To determine the cube of a number within the primary scaling range, that number scaling is located on the $N_1$ scale, interpolating as necessary. Thus, determining $4.6^3$, the value opposite 4.6 on the $N_1$ scale appears on the $N_1{}^3$ scale as about 97.2 (true value 97.336). Conversely, to determine a cube root, the given number is located on the $N_2$ scale, and the root is read on the $N_2{}^{1/3}$ scale. Thus, solving $35^{1/3}$, the value opposite 35 on the $N_2$ scale is, on the $N_2{}^{1/3}$ scale, 3.27 (true value 3.2710).

Values beyond the primary scalings are broken down into factors, so, for instance, the value of $1{,}500^{1/3}$ is equal to $1{,}000^{1/3} \times 1.5^{1/3}$. The cube roots of the factors are read as 10.0 and 1.145, respectively, so $1{,}500^{1/3} = 11.45$ (true value 11.447). Conversely, $27^3 = 10^3 \times 2.7^3 = 1{,}000 \times 19.67 = 19{,}670$ (true value 19,683).

The same principle applies even when three or more factors are required to break down the figures to the limits of the chart.


(100) [1.0]
(1,000,000) [1.0]
N
$N^3$
$N^{\frac{1}{3}}$
N
(10) [0.1]
(1,000) [0.001]

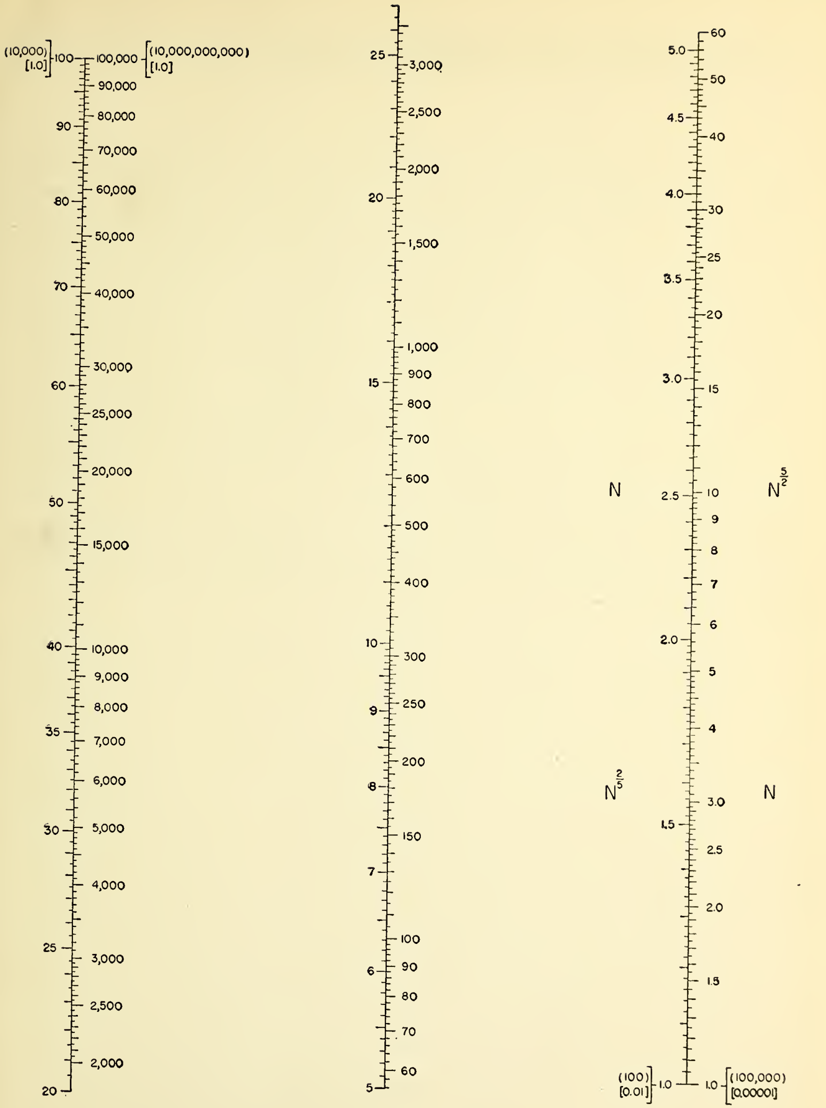
# 4. $\frac{5}{2}$ and $\frac{2}{5}$ Powers

**Method for the determination** of those functions. Tables for these functions are not readily available, and even when they are, computations of proportional differences, for interpolation, will be necessary more often than not. Where tables are not available, solution becomes a series of computations involving determinations of logarithms and antilogarithms.

**Procedure.** To determine the $\frac{5}{2}$ power of a number within the primary scaling range, that number scaling is located on the $N_1$ scale, interpolating as necessary. Thus, determining $65.0^{5/2}$, the value opposite 65.0 on the $N_1$ scale appears on the $N_1^{5/2}$ scale as 34,100 (true value 34,062). Conversely, to determine a $\frac{2}{5}$ power, the given number is located on the $N_2$ scale, and the result is read on the $N_2^{2/5}$ scale. Thus, solving $3{,}125^{2/5}$, the value opposite 3,125 on the $N_2$ scale is, on the $N_2^{2/5}$ scale, 25.0, the true value.

Values beyond the primary scalings are broken down into factors, so, for instance, the value of $220^{5/2}$ is equal to $100^{5/2} \times 2.2^{5/2}$. The $\frac{5}{2}$ powers of the factors are read as 100,000 and 7.18, so the result is 718,000 (true value 717,890). Conversely, $0.59^{2/5}$ is equal to $0.00001^{2/5} \times 59{,}000^{2/5}$. The $\frac{2}{5}$ powers of the factors are read as 0.01 and 81, respectively, and the final result is 0.81.

The same principle applies even when three or more factors are required to break down the figures to the limits of the chart.


N
N^5/2
N^2/5
N
(10,000) [1.0] 100
100,000 (10,000,000,000) [1.0]
(100) [0.01] 1.0
1.0 (100,000) [0.00001]

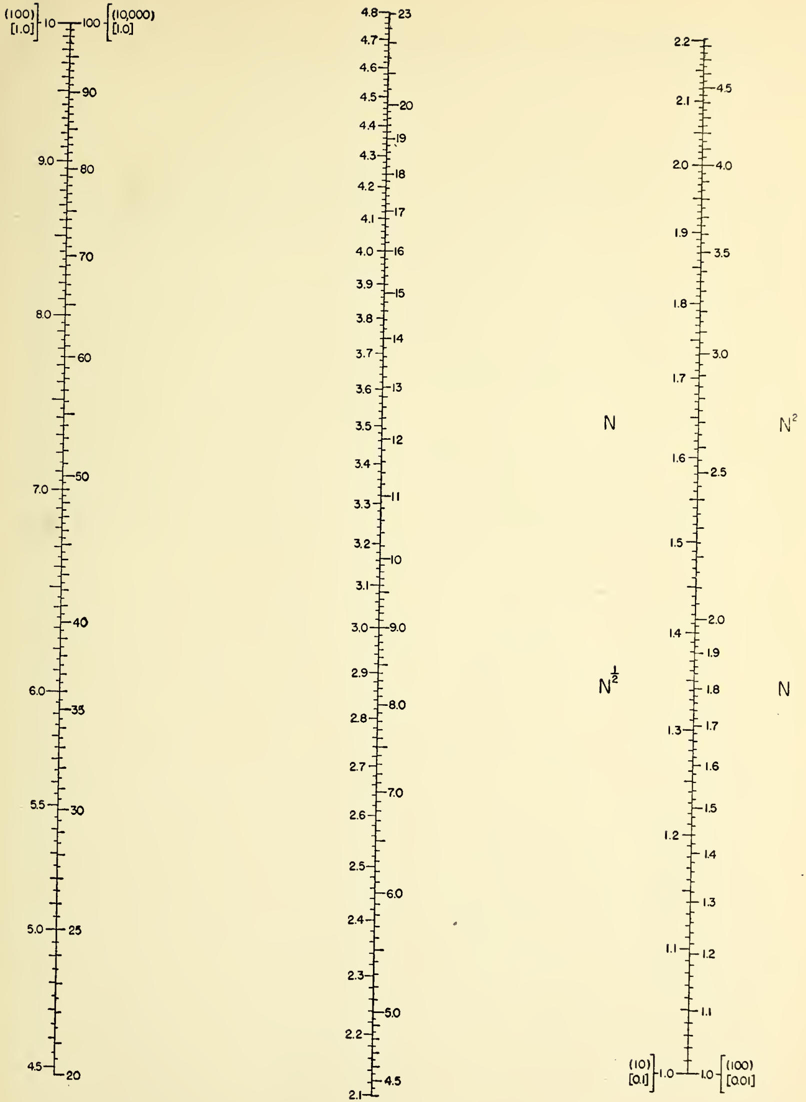
# 5. Squares and Square Roots

**Method for the determination** of those functions. Tables for these functions are more or less readily available, it is true, but even so, a computation of proportional differences, for interpolation, is necessary more often than not. Graphical interpolation is far simpler and quicker than numerical; for that reason, as well as to bring such functional determinations within one set of covers, this chart is included.

**Procedure.** To determine the square of a number within the primary scaling range, the number scaling is located on the $N_1$ scale, interpolating as necessary. Thus, determining $7.5^2$, the value opposite 7.5 on the $N_1$ scale appears on the $N_1{}^2$ scale as 56.2+ (true value 56.25). Conversely, to determine a square root, the given number is located on the $N_2$ scale, and the root is read on the $N_2{}^{1/2}$ scale. Thus solving $\sqrt{50}$, or $50^{1/2}$, the value opposite 50 on the $N_2$ scale is, on the $N_2{}^{1/2}$ scale, 7.075 (true value 7.071).

Values beyond the primary scalings are broken down into factors, so, for instance, the value of $500^{1/2}$ is equal to $100^{1/2} \times 5.0^{1/2}$. The square roots of the factors are read as 10.0 and 2.238, respectively, so the result is 22.38 (true value 22.36). Conversely, $21^2 = 10^2 \times 2.1^2 = 100 \times 4.41 = 441$, the true value.

The same principle applies even when three or more factors are required to break down the figures to the limits of the chart.

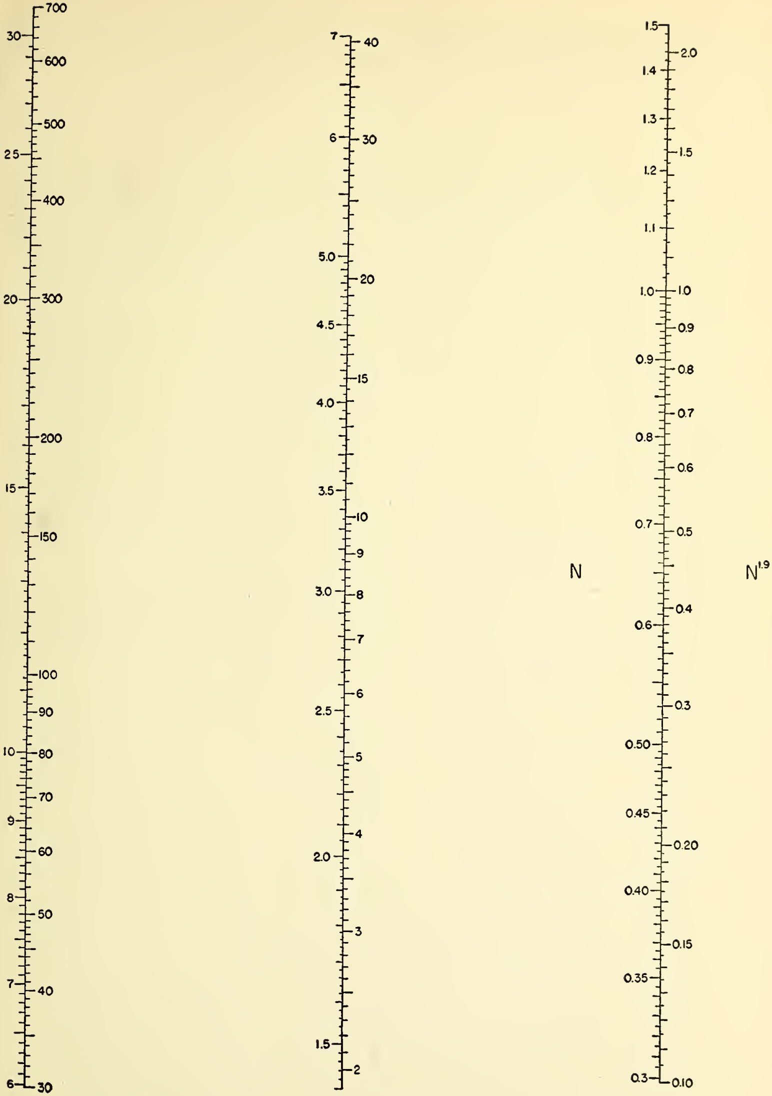

(100) [1.0] 10
100 (10,000) [1.0]
4.5 20
N
N²
N½
N
(10) [0.1] 1.0
1.0 (100) [0.01]

# 6. 1.9 Power

**Method for the determination** of that function, used in various formulas for the flow friction in pipe lines. Tables for this function are not readily available, and even when they are, computations of proportional differences, for interpolation, will be necessary more often than not. Where tables are not available, solution becomes a series of computations involving determinations of logarithms and antilogarithms.

**Procedure.** The primary scaling of the chart is thought to cover the maximum range of the quantities involved, and to find the 1.9 power of a number, that number scaling is located on the $N$ scale, interpolating as necessary. Thus, determining $1.4^{1.9}$, the value opposite 1.4 on the $N$ scale appears on the $N^{1.9}$ scale as 1.90 (true value 1.895).

Although it is felt that there is little use of this function beyond the limits of the scaling, the chart can yet be used for the exceptional cases. It is necessary only to apply the principle explained in the remarks introductory to this group of functional charts, breaking down the figures into factors which *do* fall within the chart scaling limits. The final answer is then the product of the 1.9 power of the factors.



30
25
20
15
10
9
8
7
6
700
600
500
400
300
200
150
100
90
80
70
60
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40
30
7
6
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
40
30
20
15
10
9
8
7
6
5
4
3
2
N
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.50
0.45
0.40
0.35
0.3
2.0
1.5
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.20
0.15
0.10
N1.9

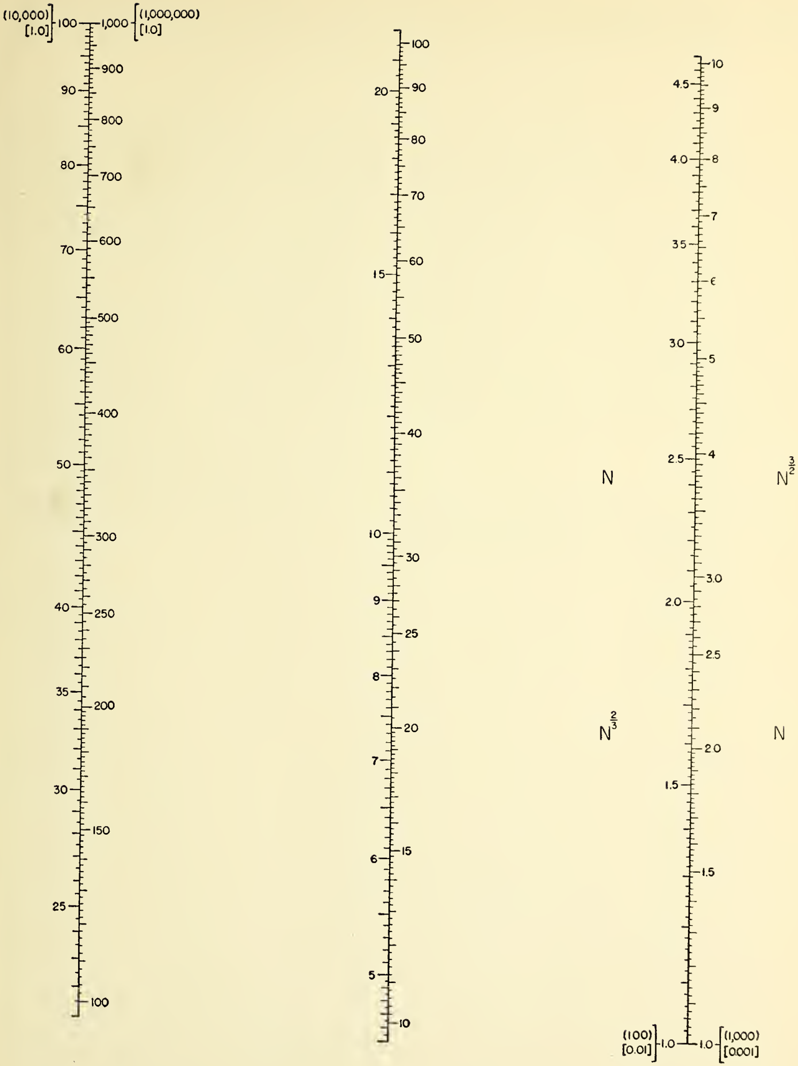
# 7. $^3/_2$ and $^2/_3$ Powers

**Method for the determination** of those functions. Tables for these functions are not readily available, and even when they are, computations of proportional differences, for interpolation, will be necessary more often than not. Where tables are not available, the use of square and cube tables in combination is possible, but still with the necessary computations for interpolation, as well as the added worry of the decimal point. Where no tables are available, solution becomes a series of computations involving determination of logarithms and antilogarithms.

**Procedure.** To determine the $^3/_2$ power of a number within the primary scaling range, that number scaling is located on the $N_1$ scale, interpolating as necessary. Thus, determining $21.0^{3/2}$, the value opposite 21.0 on the $N_1$ scale appears on the $N_1{}^{3/2}$ scale as 96.0 (true value 96.234). Conversely, to determine a $^2/_3$ power, the given number is located on the $N_2$ scale, and the result is read on the $N_2{}^{2/3}$ scale. Thus, solving $125.0^{2/3}$, the value opposite 125.0 on the $N_2$ scale is, on the $N_2{}^{2/3}$ scale, 25.0, the true value.

Values beyond the primary scalings are broken down into factors, so, for instance, the value of $150^{3/2}$ is equal to $100^{3/2} \times 1.5^{3/2}$. The $^3/_2$ powers of the factors are read as 1,000 and 1.837, respectively, so the result is 1,837 (true value 1,837.1). Conversely, $0.512^{2/3}$ is equal to $0.001^{2/3} \times 512^{2/3}$. These $^2/_3$ powers are read as 0.01 and 64.0, respectively, and the final result is 0.64, the true value.

The same principle applies even when three or more factors are required to break down the figure to the limits of the chart.

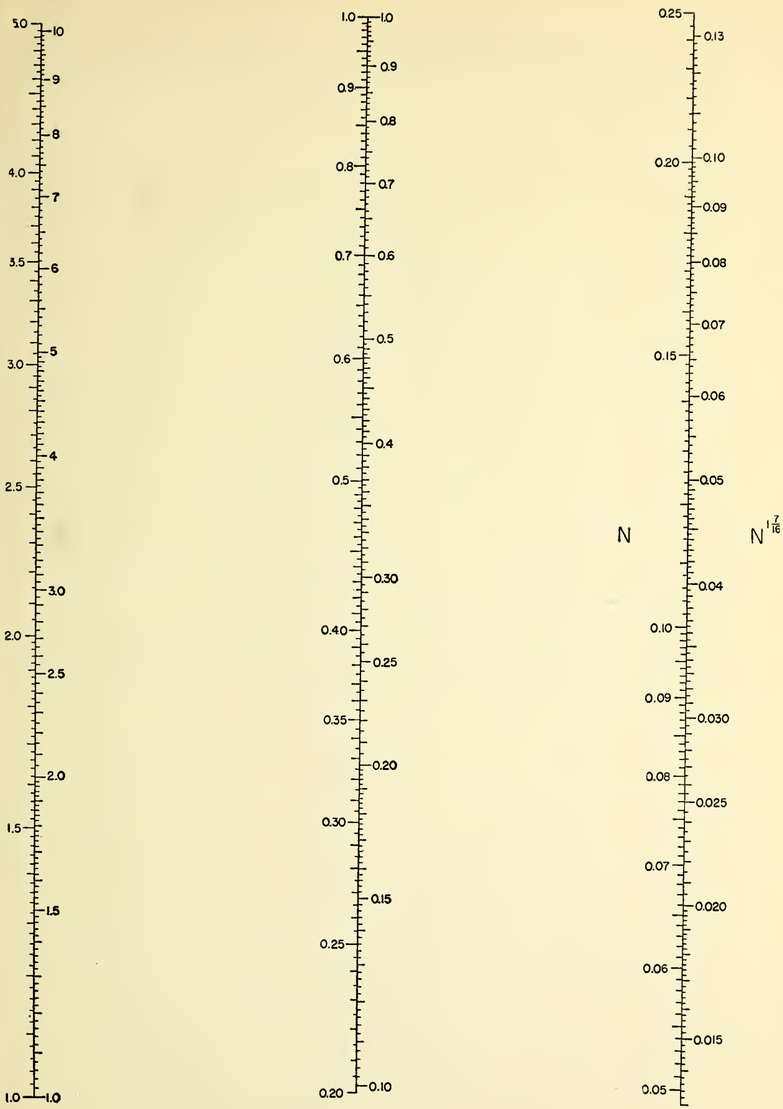

(10,000)
[1.0]
(1,000,000)
[1.0]
N
N^(3/2)
N^(2/3)
N
(100)
[0.01]
(1,000)
[0.001]

# 8. $1\frac{7}{16}$ Power

**Method for the determination** of that function, used in a recently proposed substitute for the Francis formula for determining the discharge of water over weirs with end contractions. Tables for this function have never come to the attention of this writer, but even if they are available, computations of proportional differences, for interpolation, will be necessary more often than not. With tables not available, solution becomes a series of computations involving determinations of logarithms and antilogarithms.

**Procedure.** The primary scaling of the chart is thought to cover the maximum range of the quantities involved, and to find the $1\frac{7}{16}$ power of a number, that number scaling is located on the *N* scale, interpolating as necessary. Thus, determining $0.50^{1.4375}$, the value opposite 0.50 on the *N* scale appears on the opposite scale as 0.369 (true value 0.3681).

Although it is felt that there is little use of this function beyond the limits of the scaling, the chart can yet be used for the exceptional cases. It is necessary only to apply the principle explained in the remarks introductory to this group of functional charts, breaking down the figures into factors which *do* fall within the chart scaling limits. The final answer is then the product of the $1\frac{7}{16}$ power of the factors.


N
$N^{1\frac{7}{16}}$

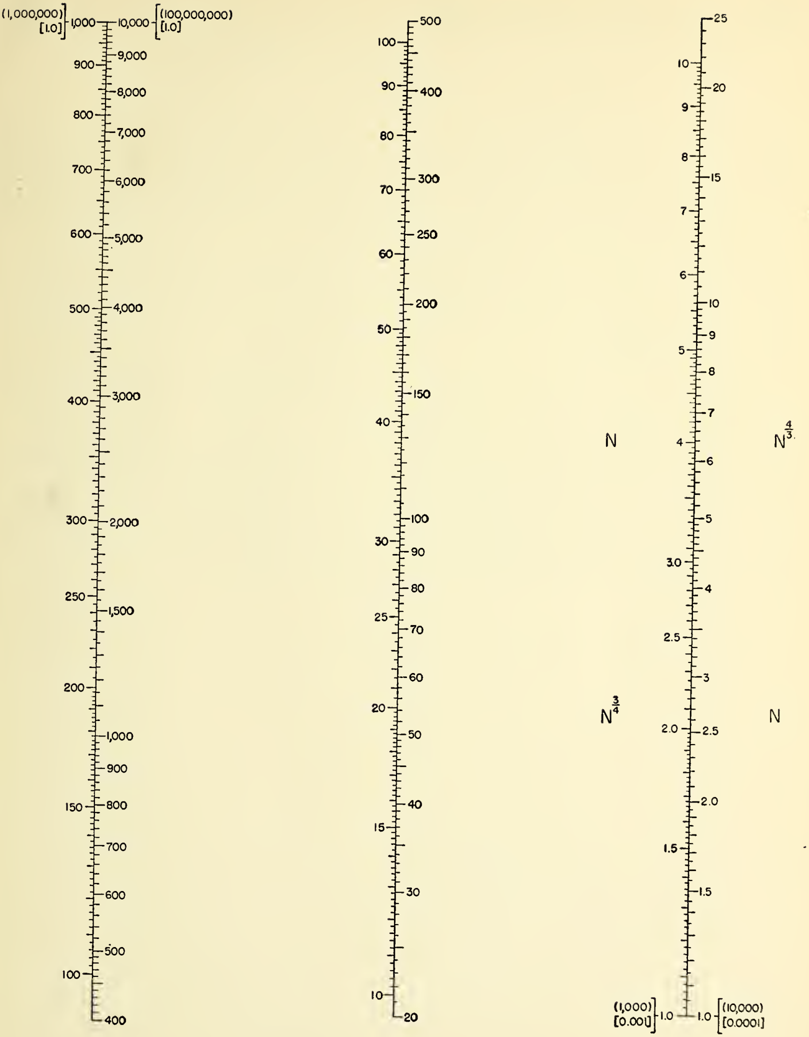
# 9. $^4/_3$ and $^3/_4$ Powers

**Method for the determination** of those functions. Tables for these functions are not readily available, and even when they are, computations of proportional differences, for interpolation, will be necessary more often than not. Where tables are not available, the use of cube tables and double use of square tables in combination *is* possible, though tedious, but still with the necessary computations for interpolation, as well as the added worry of the decimal point. Where no tables are available, solution becomes a series of computations involving determinations of logarithms and antilogarithms.

**Procedure.** To determine the $\frac{4}{3}$ power of a number within the primary scaling range, that number scaling is located on the $N_1$ scale, interpolating as necessary. Thus, determining $3.375^{4/3}$, the value opposite 3.375 on the $N_1$ scale appears on the $N_1^{4/3}$ scale as 5.08 (true value 5.0625). Conversely, to determine a $\frac{3}{4}$ power, the given number is located on the $N_2$ scale, and the result is read on the $N_2^{3/4}$ scale. Thus, solving $500^{3/4}$, the value opposite 500 on the $N_2$ scale is, on the $N_2^{3/4}$ scale, 105.7 (true value 105.73).

Values beyond the primary scalings are broken down into factors, so, for instance, the value of $27{,}000^{4/3}$ is equal to $1{,}000^{4/3} \times 27.0^{4/3}$. The $\frac{4}{3}$ powers of the factors are read as 10,000 and 81.0, respectively, so the result is 810,000, the true value. Conversely, $0.5^{3/4}$ is equal to $0.0001^{3/4} \times 5{,}000^{3/4}$. These $\frac{3}{4}$ powers are read as 0.001 and 595, respectively, and the final result is 0.595 (true value 0.59468).

The same principle applies even when three or more factors are required to break down the figure to the limits of the chart.



(1,000,000) [1.0] 1,000
10,000 [(100,000,000) [1.0]
N
$N^{\frac{3}{4}}$
$N^{\frac{4}{3}}$
N
(1,000) [0.001] 1.0
1.0 [(10,000) [0.0001]

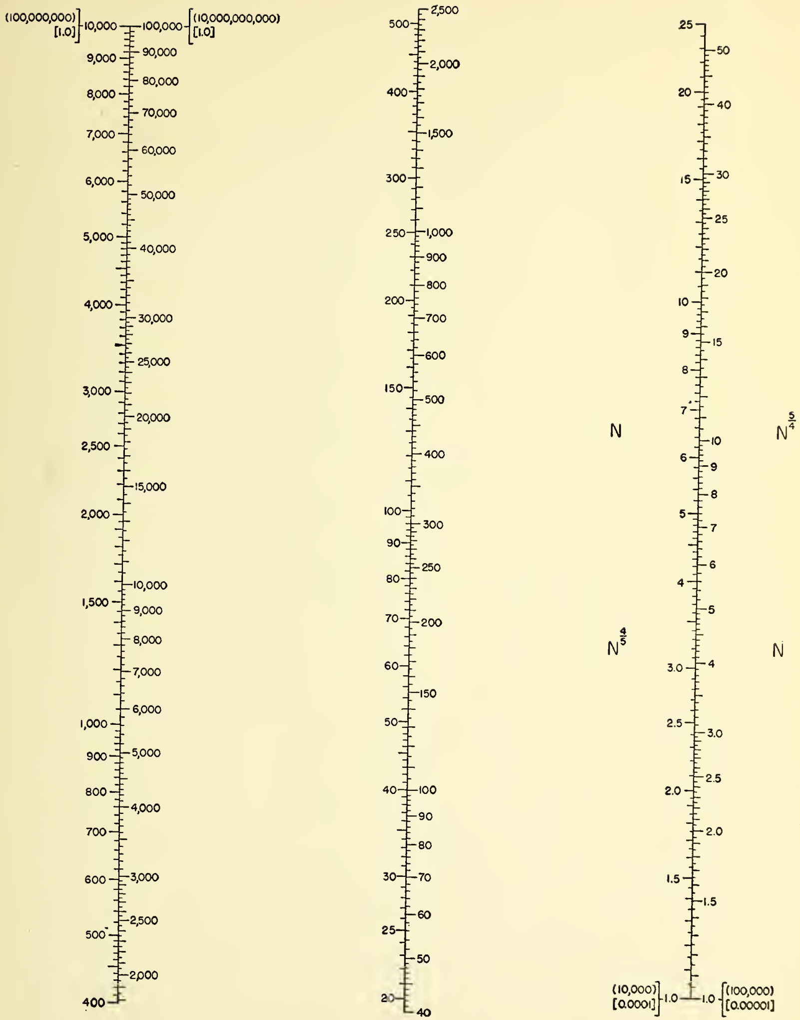
# 10. $^5/_4$ and $^4/_5$ Powers

**Method for the determination** of those functions. Tables for these functions are not readily available, and even when they are, computations for proportional differences, for interpolation, will be necessary more often than not. Where tables are not available, solution becomes a series of computations involving determinations of logarithms and antilogarithms.

**Procedure.** To determine the $5/4$ power of a number within the primary scaling range, that number scaling is located on the $N_1$ scale, interpolating as necessary. Thus, determining $550^{5/4}$, the value opposite 550 on the $N_1$ scale appears on the $N_1{}^{5/4}$ scale as 2,660 (true value 2,664). Conversely, to determine a $4/5$ power, the given number is located on the $N_2$ scale, and the result is read on the $N_2{}^{4/5}$ scale. Thus, solving $2{,}700^{4/5}$, the value opposite 2,700 on the $N_2$ scale is, on the $N_2{}^{4/5}$ scale, 557 (true value 556).

Values beyond the primary scalings are broken down into factors, so, for instance, the value of $150{,}000^{5/4}$ is equal to $10{,}000^{5/4} \times 15.0^{5/4}$. The $5/4$ powers of the factors are read as 100,000 and 29.35, respectively, so the result is 2,935,000 (true value 2,952,-000$\pm$). Conversely, $0.07775^{4/5}$ is equal to $0.00001^{4/5} \times 7{,}775^{4/5}$. These $4/5$ powers are read as 0.0001 and 1,300, respectively, and the final result is 0.1300 (true value 0.1296).

The same principle applies even when three or more factors are required to break down the figure to the limits of the chart.



N
$N^{\frac{4}{5}}$
$N^{\frac{5}{4}}$
N
(100,000,000)
[1.0]
(10,000,000,000)
[1.0]
(10,000)
[0.0001]
(100,000)
[0.00001]

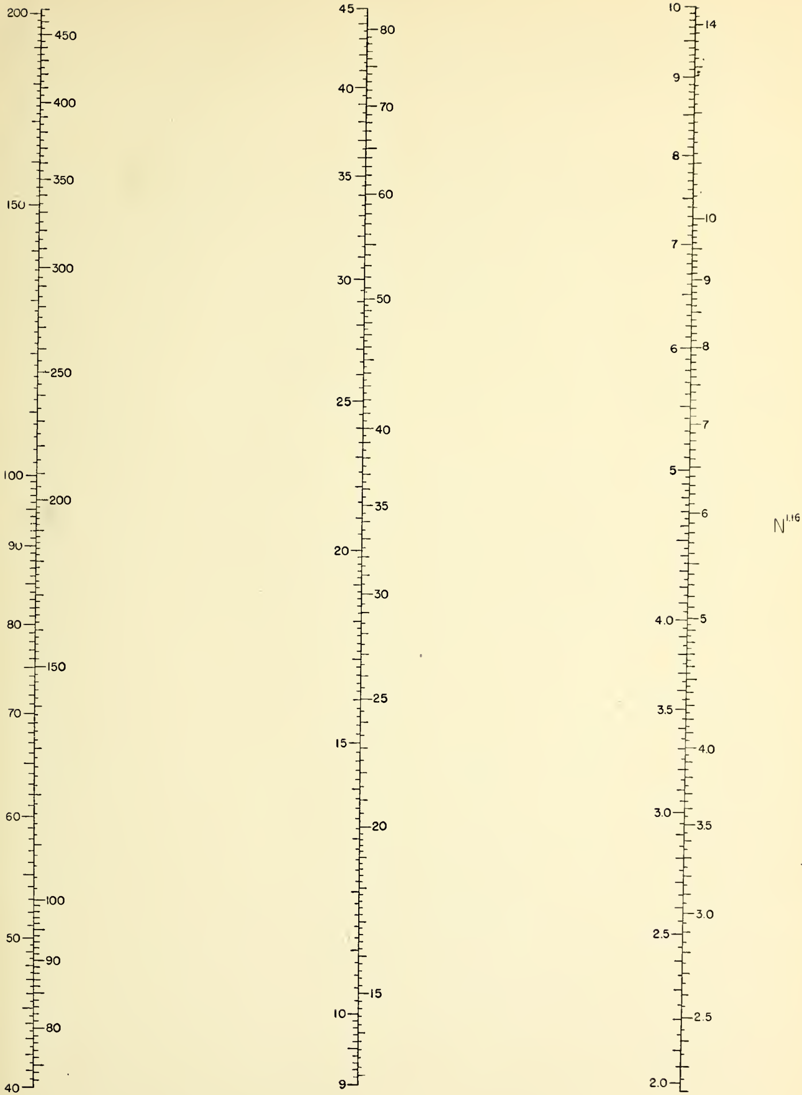
# 11. 1.16 Power

**Method for the determination** of that function, used in certain formulas for the determination of flow friction in hydraulic flow lines. Tables for this function have never come to the attention of this writer, but even if they are available, computations of proportional differences, for interpolation, will be necessary more often than not. With tables not available, solution becomes a series of computations involving determinations of logarithms and antilogarithms.

**Procedure.** The scaling of the chart is thought to cover the maximum range of the quantities involved, with but one exception noted later. To find the 1.16 power of a number, that number scaling is located on the $N$ scale, interpolating as necessary. Thus, determining $9.5^{1.16}$, the value opposite 9.5 on the $N$ scale appears on the opposite scale as 13.62, the true value.

The use of this function in the formulas mentioned is as the function of the flow-conduit diameter, and when the unit is inches, the chart should cover practically all cases. If the figure used is beyond the chart scaling, recourse may be had to the principle explained in the remarks introductory to this group of functional charts, breaking down the figure into factors which *do* fall within the chart scaling limits. The final answer is then the product of the 1.16 powers of the factors.

Should the particular formula used be based upon units of feet instead of inches, the 1.16 power of the number must be multiplied by the 1.16 power of $\frac{1}{12}$, which is 0.05472.


200 150 100 90 80 70 60 50 40
450 400 350 300 250 200 150 100 90 80
45 40 35 30 25 20 15 10 9
80 70 60 50 40 35 30 25 20 15
10 9 8 7 6 5 4.0 3.5 3.0 2.5 2.0
14 10 9 8 7 6 5 4.0 3.5 3.0 2.5
$N^{1.16}$

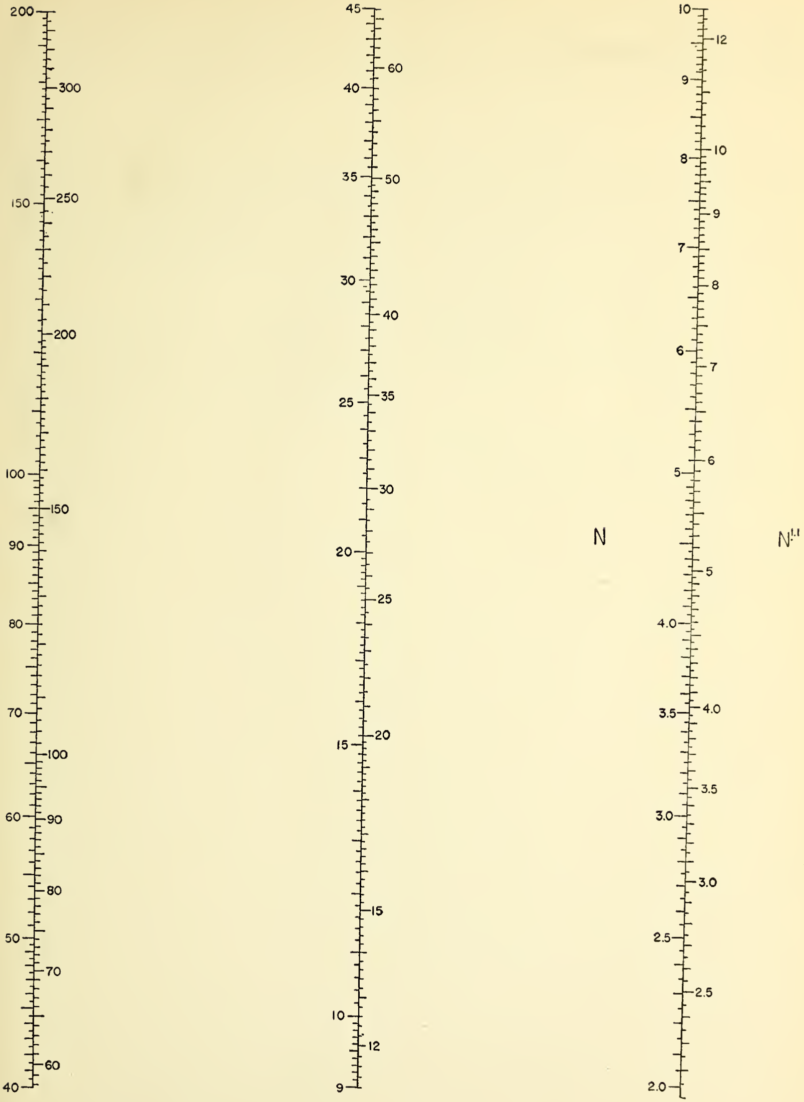
# 12. 1.1 Power

**Method for the determination** of that function, used in certain formulas for the determination of flow friction in hydraulic flow lines. Tables for this function are not readily available, and even when they are, computations of proportional differences, for interpolation, will be necessary more often than not. Where tables are not available, solution becomes a series of computations involving determinations of logarithms and antilogarithms.

**Procedure.** The scaling of the chart is thought to cover the maximum range of the quantities involved, with but one exception noted later. To find the 1.1 power of a number, that number scaling is located on the *N* scale, interpolating as necessary. Thus, determining $20^{1.1}$, the value opposite 20 on the *N* scale appears on the opposite scale as 27.05 (true value 26.985).

The use of this function in the formulas mentioned is as the function of the flow-conduit diameter, and when the unit is inches, the chart should cover practically all cases. If the figure used is beyond the chart scaling, recourse may be had to the principle explained in the remarks introductory to this group of functional charts, breaking down the figure into factors which *do* fall within the chart scaling limits. The final answer is then the product of the 1.1 powers of the factors.

Should the particular formula used be based upon units of feet instead of inches, the 1.1 power of the number must be multiplied by the 1.1 power of $\frac{1}{12}$, which is 0.065.

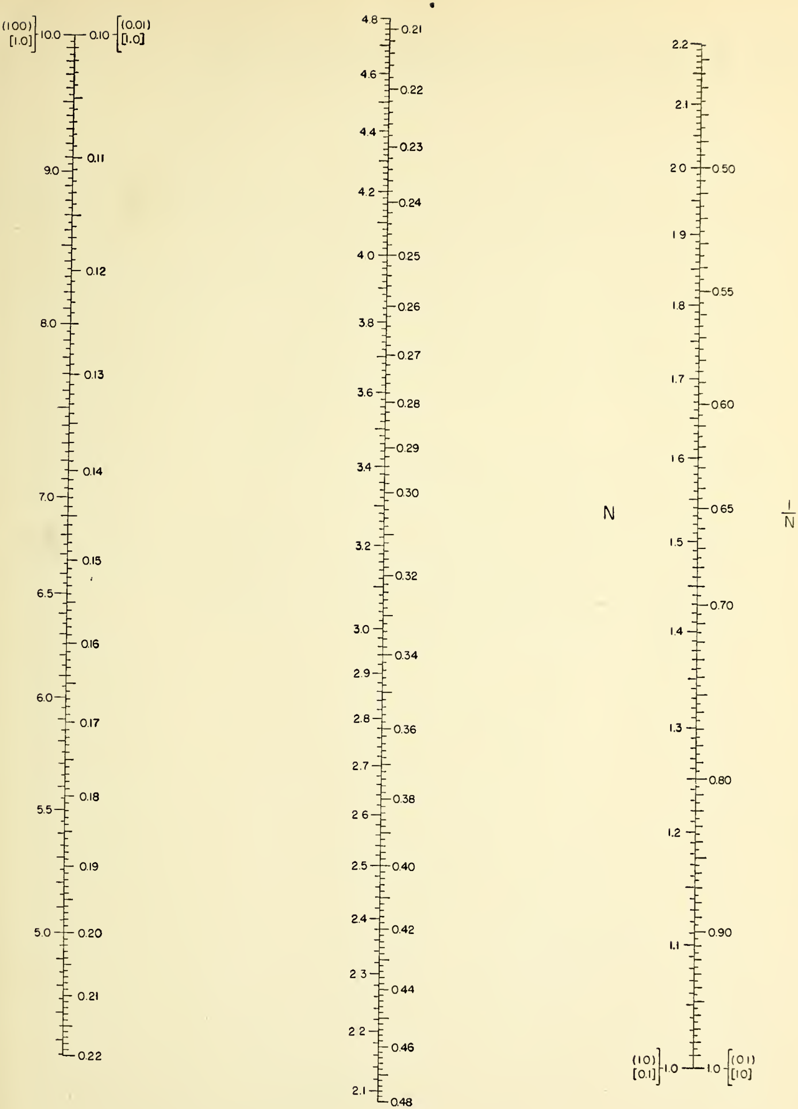

N
$N^{1.1}$

# 13. Reciprocals

**Method for the determination** of those functions. Tables are readily available, it is true; but even so, a computation of proportional differences, for interpolation, is necessary more often than not. However, graphical interpolation is far simpler and quicker than numerical, so for that reason, as well as to bring such functional determinations within one set of covers, this chart is included.

**Procedure.** To determine the reciprocal of a number within the scaling range, the number scaling is located, interpolating as necessary, and the reciprocal is read on the opposite scale. Thus, determining 1/2.1, the value opposite 2.1 on the scale is read on the opposite side as 0.476 (true value 0.4762).

For determination of reciprocals beyond the scaling limits, the decimal point is merely moved equal distances in opposite directions for number and reciprocal. Thus, 1/210 is 0.00476, and 1/0.0021 is 476.


(100) [1.0] 10.0
0.10 (0.01) [1.0]
9.0 8.0 7.0 6.5 6.0 5.5 5.0
0.11 0.12 0.13 0.14 0.15 0.16 0.17 0.18 0.19 0.20 0.21 0.22
4.8 4.6 4.4 4.2 4.0 3.8 3.6 3.4 3.2 3.0 2.9 2.8 2.7 2.6 2.5 2.4 2.3 2.2 2.1
0.21 0.22 0.23 0.24 0.25 0.26 0.27 0.28 0.29 0.30 0.32 0.34 0.36 0.38 0.40 0.42 0.44 0.46 0.48
N
2.2 2.1 2.0 1.9 1.8 1.7 1.6 1.5 1.4 1.3 1.2 1.1
(10) [0.1] 1.0
0.50 0.55 0.60 0.65 0.70 0.80 0.90
1.0 (0.1) [10]
1/N

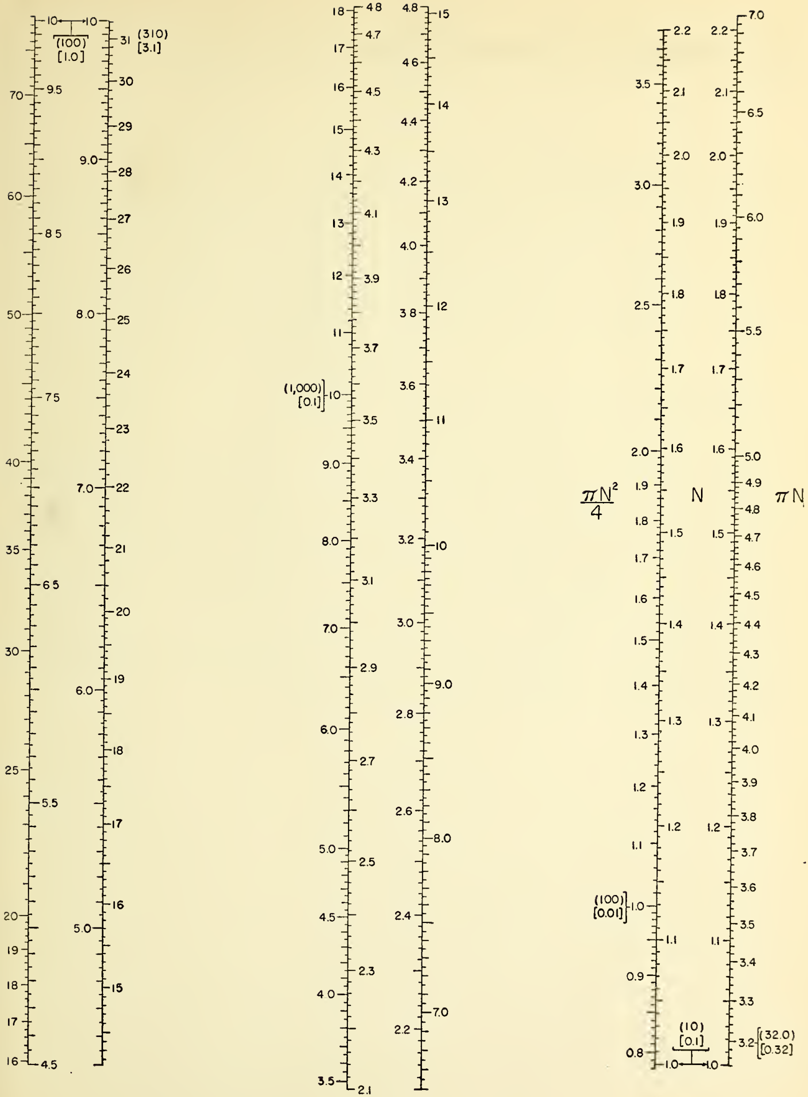
# 14. Circle Circumference and Area

**Method for the determination** of those functions. Tables are readily available, it is true; but even so, a computation of proportional differences, for interpolation, is necessary more often than not. However, graphical interpolation is far simpler and quicker than numerical, so for that reason, as well as to bring such functional determinations within one set of covers, this chart is included.

**Procedure.** To determine the area or circumference of a circle within the limits of the chart scaling, the value of the diameter is located on the $N$ scale, interpolating as necessary. On the opposite side of the scale may then be read the area (on the $\pi N^2/4$ scale) or the circumference (on the $\pi N$ scale). Thus, for a diameter of 2.16, the area is read as 3.67 (true value 3.6644) and the circumference as 6.78 (true value 6.7858).

For determining values beyond the scale limits, the only adjustment necessary is, of course, in the decimal point. If the figure for diameter has the decimal point moved any number of places, in either direction, the figure for circumference ($\pi N$) will have the decimal point moved the same amount and in the same direction. The figure for area ($\pi N^2/4$) will have its decimal point moved *twice* as many places in the same direction.

Thus, with diameter 216, the circumference is 678 and the area 36,700, and for diameter 0.216 the circumference is 0.678 and the area is 0.0367.


(100)
[1.0]
(310)
[3.1]
(1,000)
[0.1]
$\frac{\pi N^2}{4}$
N
$\pi N$
(100)
[0.01]
(10)
[0.1]
(32.0)
[0.32]

# 15. Weight of Steel Plate

**Method for the determination** of the weight of steel plates of any thickness and for the thickness gauges in most common use. Such data are easily available in tabular form, but the chart has proved so convenient and helpful in the past that its being included with the other charts seems justified.

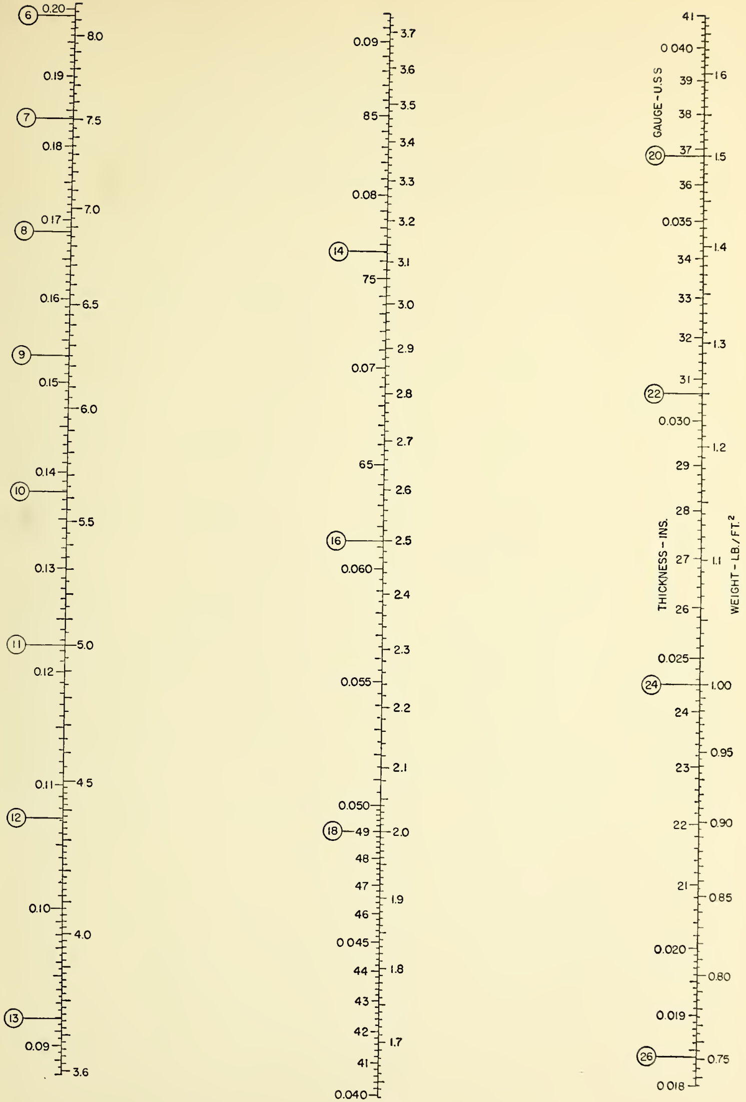
**Procedure.** To use the chart, the thickness in inches is located on the left-hand scaling, interpolating as necessary. The weight per square foot is then read on the opposite side of the scale. Where thickness is given by U.S.S. gauge number, the left-hand side of the scale is located by the circled figures, the gauge numbers. The thickness may be simultaneously read on the primary left-hand scaling.

For thickness values beyond the scale limits, it is necessary only to multiply both thickness and weight figures by the same multiple of 10. Naturally, the gauge designations do not apply in such cases.

Illustrating the use of the chart, to find the properties of plate of No. 20 U.S.S. gauge, locate the scaling, circled, for No. 20, on the left-hand side of the scale. This is at scaling 0.0368 (the true value), as the thickness in inches. On the opposite side of the scale the weight is read as 1.500 pounds per square foot (also the true value).

In case the thickness value is beyond the scale, as 1.25 inches, find the scaling on the left-hand side for 0.125, opposite which is read a weight of 5.10 pounds per square foot. Since the given thickness was ten times the scale value used, the weight will be ten times that read, or 51.0 pounds per square foot.

Values used were as given by the Bethlehem Steel Company in its *Steel Plate Handbook*.


GAUGE - U.S.S
THICKNESS - INS.
WEIGHT - LB./FT.²

# Group II
# GENERAL CHARTS

This group of charts is gathered into one series, as being, in addition to the functional scale charts preceding, of rather general use in many fields of engineering. This group differs from the first one, however, in that where the first group included single, though broken, simple scales, this second group brings the first charts dealing with multiple variables and introduces the alignment type of nomograph.

In all of the earlier charts, the scalings were so made that the quantities which bore the represented relationship coincided in a distance along the base line. In the charts to follow, however, treating of more than two quantities at a time, the design is such that, for any stage of solution (and in some there may be several stages), the scale points concerned are in a space relationship rather than that of a single line.

In some charts, the purely alignment type, the scale values concerned in any single operation stage will be on a straight line intersecting the various scales. In other charts, of the intersection type, the conditions bearing the proper relationship in any stage of operation will be represented by a single point in the chart, which point may give the measure of the various related quantities by the scaling of rectangular coordinates (distorted when required) and/or by other families of lines, parallel, radial, or families of curves, or various combinations of these. Further, there are some charts where, in the interest of greater accuracy or greater utility, a combination of alignment and intersection types has been used.

In each case, the chart is explained, with specimen computations, and in most cases a key diagram is given to illustrate the sequence of operations.

## Charts in Group II

16. Reciprocals of Reciprocal Sums
17. Compound Interest Amount
18. Sinking Fund Deposits
19. Present Value of Annuity
20. Present Value of Future Amount
21. Properties of Rectangles
22. Area of Isosceles Trapezoids
23. Properties of Right Triangles

# 16. Reciprocals of Reciprocal Sums

**Method for the determination** of the reciprocal of the sum of a series of reciprocals. It solves equations of the form

$$X = \frac{1}{(1/a) + (1/b) + (1/c) + (1/d) + \cdots}$$

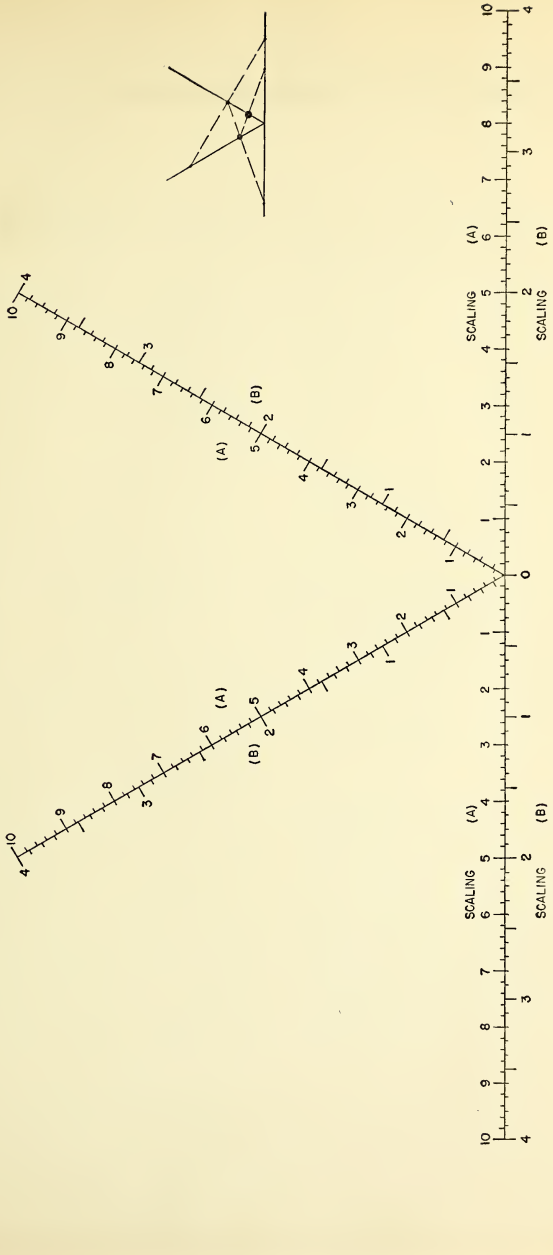
This equation applies for finding the equivalent electrical resistance of a number of separate resistances in parallel, where $X$ is the equivalent resistance and $a$, $b$, $c$, and $d$ are values of the individual paralleled resistances. The equivalent effect of electrical capacitances *in series* is determined by similar means.

For those who have occasion to make up nomographic charts for their own use, this general equation finds application in those alignment-type nomographs of the simple addition (including logarithmic) type. Here the scaling unit on the scale of sums is equal to the reciprocal of the sum of the reciprocals of the scaling units of the original scales. More simply, for a chart for $X = Y + Z$, where a unit on the $Y$ scale has length $y$ and a unit on the $Z$ scale has length $z$, the length of a unit on the $X$ scale $= x = \dfrac{1}{(1/y) + (1/z)}$.

**Procedure.** The chart is in the form of four radiating scaled lines, and in each operation the two starting points are on *alternate* rays, and where more than two original terms are used, the start is made with any two such alternate scales. Each scale has two scaling units, and the same scaling, $A$ or $B$ as the case may be, must be used consistently.

To illustrate operation, assume but two values, 2 and 3, as terms $a$ and $b$ (see first paragraph). A straight line (straightedge) intersecting one of the scales at 2 and the second-removed scale at 3 intercepts the intermediate scale at 1.2, the true value. For the sum of $\frac{1}{2}$ and $\frac{1}{3}$ is $\frac{5}{6}$, whose reciprocal is $\frac{6}{5}$, or 1.2.

Illustrating the use of three original quantities, take $a = 9$, $b = 8$, and $c = 7$. A straight line is run (straightedge) from 9 on one scale to 8 on the one second removed, and its intersection on the intermediate scale is noted. This scale point is then in turn connected by a straight line to the scale value for 7 on the scale second removed from it, and the final result is read on the scale intermediate to *these* two scales, using the same group of scales throughout, the $A$ scales in this case. The result appears as about 2.64, where numerical computation gives the exact result as 2.6455.



SCALING (A)
SCALING (B)
(A)
(B)
SCALING (A)
SCALING (B)
(A)
(B)

# 17. Compound Interest Amount

**Method for the determination** of the total amount of principal and interest, when a principal is placed at compound interest for various periods, at various rates, and at various compounding intervals. The mathematical expression for the future compounded total value of a present dollar is $(1 + r)^n$, where $r$ is the interest rate, expressed as a decimal, and $n$ is the time factor.

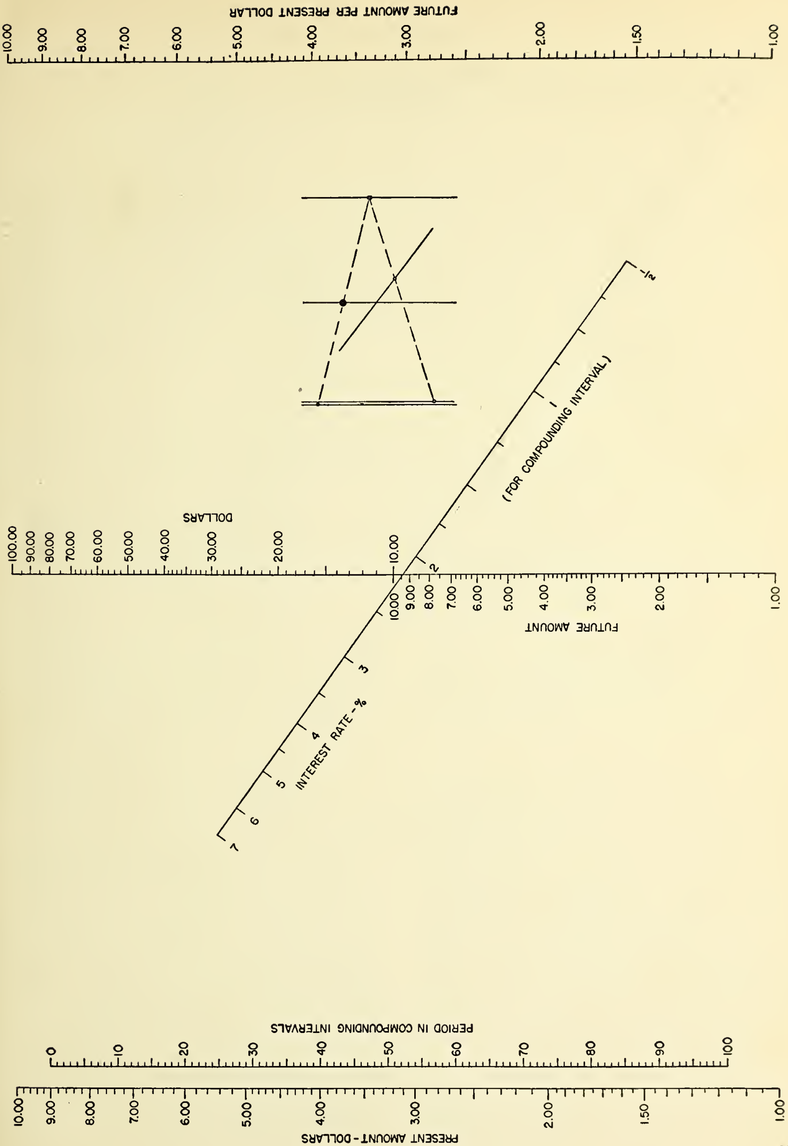
The rate is not necessarily the nominal annual rate, nor is the time factor necessarily in years. Rather, the time is measured in number of compounding intervals, and the rate as that per compounding interval. Thus, where compounding is semiannual, the time factor will be twice the number of years, and the rate will be one-half the nominal annual rate. In case of quarterly compounding, the time factor will be four times the number of years, and the rate will be one-fourth the nominal annual rate.

It is true that tables are more or less readily available for solving problems in this field, but this author has never been able to find tables which adequately cover all the conditions commonly met. Further, this chart covers fractional rates and gives more complete direct coverage of the time factor, reducing the need for interpolation. Then, too, when interpolation is necessary on the chart, it is simpler and quicker than interpolation by calculation of proportional differences.

**Procedure.** To use the chart, a straight line is projected through the scale points corresponding to time factor and interest rate, and the future value per dollar is read where this line crosses the right-hand scale. A straight line from this point to the extreme left-hand scale, for actual present amount, will then intersect the center scale at a point giving the total future amount. The *Present Amount* scale reads from 1.00 to 10.00 only, but for larger quantities, multiples of 10 should be used, the future amount being adjusted in the same ratio as the present amount.

Use of the chart is illustrated by determining the future value of $8,000, at 3 per cent compounded quarterly, at the end of 20 years. Twenty years is 80 compounding intervals, and the interest rate per compounding interval is ¾ per cent. A straight line through the scale point for 80 on the *Period* scale and the ¾ per cent scale point on the *Interest Rate* scale intersects the *Future Amount per Present Dollar* scale at 1.815 (true value 1.8180).

To determine the full future amount, this latter point is used with 8.00 on the *Present Amount* scale to establish the intersection on the *Future Amount* scale at 14.50, so the total future amount will be $14,500 (true value, $14,544).


FUTURE AMOUNT PER PRESENT DOLLAR
DOLLARS
FUTURE AMOUNT
INTEREST RATE - %
(FOR COMPOUNDING INTERVAL)
PERIOD IN COMPOUNDING INTERVALS
PRESENT AMOUNT - DOLLARS

# 18. Sinking Fund Deposits

**Method for the determination** of the amount of each of periodic deposits which, at various interest rates, compounding intervals, and time periods, will total desired compounded amounts. The mathematical expression for the periodic deposit required per dollar of sinking fund is $1/[(1+r)^n-1]$, where $r$ is the interest rate on deposits, expressed as a decimal, and $n$ is the time factor.

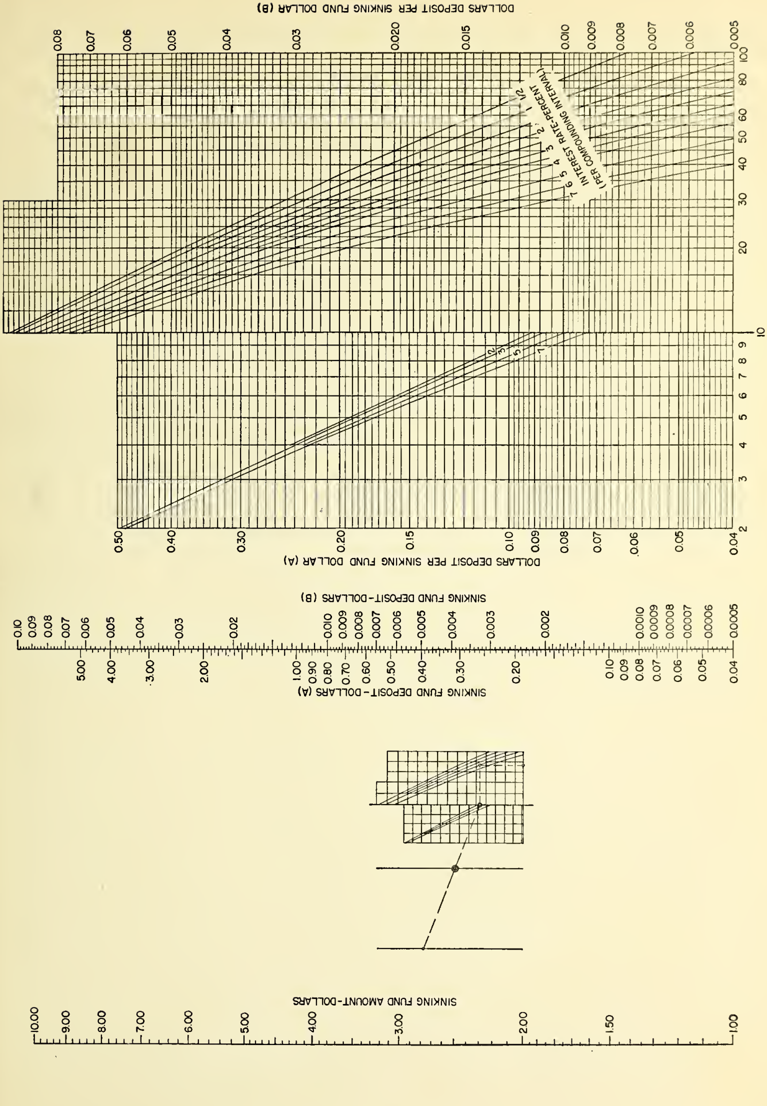
The rate is not necessarily the nominal annual rate, nor is the time factor necessarily in years. Rather, the time is measured in number of compounding intervals and the rate as that per compounding interval. Thus, where compounding is semiannual, the time factor will be twice the number of years, and the rate will be one-half the nominal annual rate. In case of quarterly compounding, the time factor will be four times the number of years, and the rate will be one-fourth the nominal.

It is true that tables are more or less readily available for solving problems in this field, but this author has never been able to find tables which adequately covered all the conditions commonly met. Further, this chart covers fractional rates and gives more complete direct coverage of the time factor, reducing the need for interpolation. Then, too, when interpolation is necessary on the chart, it is simpler and quicker than interpolation by calculation of proportional differences.

**Procedure.** To use the chart, the first stage is in the intersection-type section, where the intersection is located for the values of *Interest Rate* curves and the vertical lines of *Sinking Fund Time,* interpolating where necessary. Following horizontally across this section of the chart to the side scales will give the value of the deposit for each dollar of total sinking fund.

It should be noted that this portion of the chart is divided into two parts by the vertical line corresponding to a time factor of 10. Conditions falling on the left-hand side of this line are to be evaluated by the left-hand scale *A,* while those to the right use the right-hand scale *B.* For the second step in the use of the chart, however, the intersection point of vertical line and curve is to be carried horizontally across to the main axis, the heavy line, vertical, corresponding to a time factor of 10.

The second step in use of the chart provides for determining the periodic deposit for a fund other than \$1. This is done by use of a straight line from the intercept of the first determination on the main axis of the intersection portion of the chart, as described above, to the correct scale point, interpolating as necessary, on the *Sinking Fund Amount* scale. At the intersection of this straight line with the *Sinking Fund Deposit* scale is read the actual amount of each of the deposits to be made, using *A* or *B* scaling to correspond with the first-stage determination. The *Sinking Fund Amount* scale reads only from 1.00 to 10.00, but for larger quantities, multiples of 10 should be used, the sinking fund deposit


DOLLARS DEPOSIT PER SINKING FUND DOLLAR (B)
INTEREST RATE-PERCENT
(PER COMPOUNDING INTERVAL)
PERIOD-COMPOUNDING INTERVALS
DOLLARS DEPOSIT PER SINKING FUND DOLLAR (A)
SINKING FUND DEPOSIT-DOLLARS (B)
SINKING FUND DEPOSIT-DOLLARS (A)
SINKING FUND AMOUNT-DOLLARS

being adjusted in the same ratio as the sinking fund.

Use of the chart is illustrated by determining the necessary deposit, made during each half year, at 4 per cent interest compounded semiannually, which will make up a sinking fund of $80,000 in 25 years. Compounding being semiannual, the time is 50 compounding intervals, and the rate per compounding interval is 2 per cent. The intersection of the vertical line for 50 for *Sinking Fund Time* with the curve of 2 per cent for *Interest Rate* is read, interpolating, as 0.01175 (true value 0.01182), so the required deposit is $0.01175 per dollar of required sinking fund. This is in the *B* portion of the chart.

A straight line between this intercept on the main axis, as explained, and 8.00 on the *Sinking Fund Amount* scale, gives, on the *B* scaling on the *Sinking Fund Deposit* scale, a figure of 0.0945, so the actual deposit required, 10,000 times this, will be $945.00 (true value $945.60).

# 19. Present Value of Annuity

**Method for the determination** of the amount of the fund which, at various lengths of compounding intervals, various interest rates, and various lengths of time, will yield a required periodic return. The mathematical expression for the present value per dollar of periodic return is $[(1 + r)^n - 1]/r(1 + r)^n$, where $r$ is the interest rate on the fund, expressed as a decimal, and $n$ is the time factor.

The rate is not necessarily the nominal annual rate, nor is the time factor necessarily in years. Rather, the time is measured in number of compounding intervals, and the rate as that per compounding interval. Thus, where compounding is semiannual, the time factor will be twice the number of years, and the rate will be one-half the nominal annual rate. In case of quarterly compounding, the time factor will be four times the number of years, and the rate will be one-fourth the nominal.

It is true that tables are more or less readily available for solving problems in this field, but this author has never been able to find tables which adequately covered all the conditions commonly met. Further, this chart covers fractional rates and gives more complete direct coverage of the time factor, reducing the need for interpolation. Then, too, when interpolation is necessary on the chart, it is simpler and quicker than interpolation by calculation of proportional differences.

**Procedure.** To use the chart, the first stage is in the intersection-type section, where the intersection is located for the values of *Interest Rate* curves and the vertical lines of *Life of Annuity*, interpolating where necessary. Following horizontally across this section of the chart to the side scales will give the *Present Value* of $1 annuity, or periodic payment.

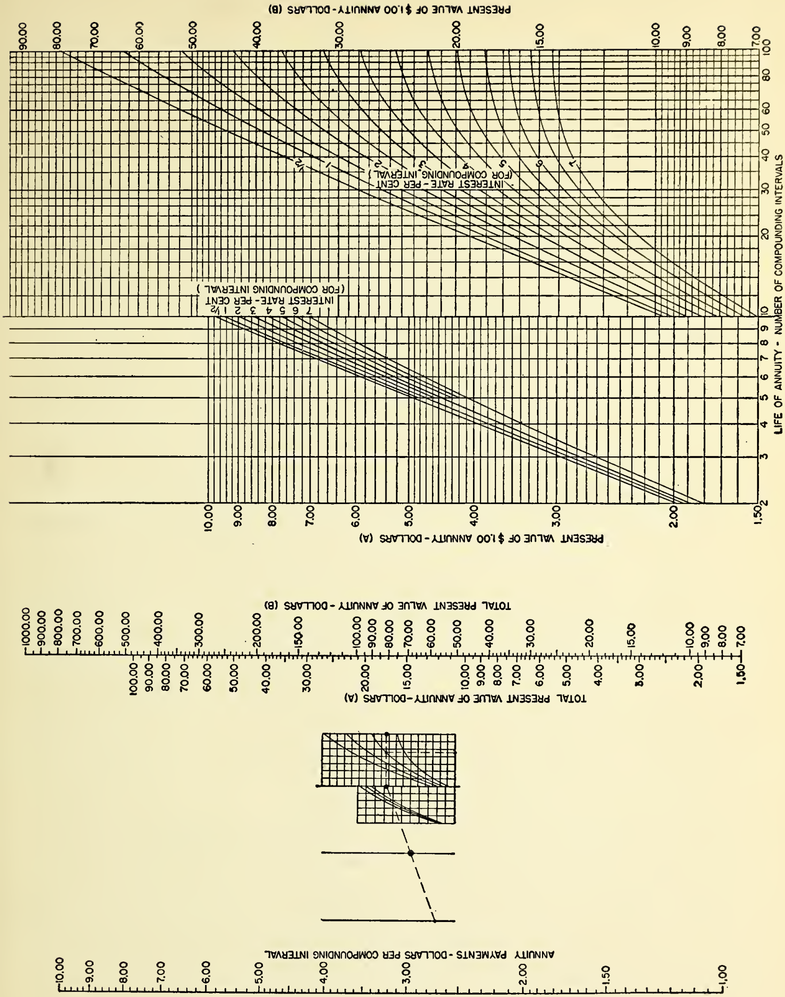
It should be noted that this portion of the chart is divided into two parts by the vertical line corresponding to a time factor of 10. Conditions falling on the left-hand side of this line are to be evaluated by the left-hand scale *A*, while those to the right use the right-hand scale *B*. For the second step in the use of the chart, however, the intersection point of vertical line and curve is to be carried horizontally across to the main axis, the heavy vertical line, corresponding to the time factor of 10.

The second step in use of the chart provides for determining the amount of the fund for an annuity, or periodic yield, other than $1. This is done by use of a straight line from the intercept of the first determination on the main axis of the intersection portion of the chart, as described above, to the correct scale point, interpolating as necessary, on the *Annuity Payments* scale. At the intersection of this straight line with the scale for *Total Present Value of Annuity* is read the amount of the required fund, using *A* or *B* scaling to correspond with the first-stage determination. The scale of *Annuity Payment* reads only from 1.00 to 10.00, but for larger quantities multiples of 10 should be used, the present value being adjusted in the same ratio as the annuity payment.

## Present Value of Annuity

Use of the chart is illustrated by determining the fund necessary to provide a quarterly payment or annuity of $1,200, with an interest rate of 2 per cent compounded quarterly, throughout a period of 15 years. With quarterly compounding, the life of the fund is 60 compounding intervals, and the rate is ½ per cent per compounding interval. The intersection of the vertical line for 60 compounding intervals with the curve for ½ per cent interest rate is read, interpolating, as 51.80 (true value 51.73), so the required fund is $51.80 per dollar of quarterly yield. This is in the *B* portion of the chart.

A straight line between this intercept on the main axis, as explained, and 1.20 on the *Annuity Payment* scale, gives, on the *B* scaling of *Total Present Value of Annuity*, a figure of 62.00, so the actual fund required is 1,000 times this, or $62,000 (true value $62,076).


PRESENT VALUE OF $1.00 ANNUITY - DOLLARS (B)
INTEREST RATE - PER CENT (FOR COMPOUNDING INTERVAL)
LIFE OF ANNUITY - NUMBER OF COMPOUNDING INTERVALS
PRESENT VALUE OF $1.00 ANNUITY - DOLLARS (A)
TOTAL PRESENT VALUE OF ANNUITY - DOLLARS (B)
TOTAL PRESENT VALUE OF ANNUITY-DOLLARS (A)
ANNUITY PAYMENTS - DOLLARS PER COMPOUNDING INTERVAL

# 20. Present Value of Future Amount

**Method for the determination** of the present worth of a future value. This is necessary in setting up a fund which, at compound interest at varying interest rates, compounding intervals, and periods of time, will amount to the required future amount. The mathematical expression for the present worth per dollar of future fund is $1/(1 + r)^n$, where $r$ is the interest rate on funds, expressed as a decimal, and $n$ is the time factor.

The rate is not necessarily the nominal annual rate, nor is the time factor necessarily in years. Rather, the time is measured in number of compounding intervals, and the rate as that per compounding interval. Thus, where compounding is semiannual, the time factor will be twice the number of years, and the rate will be one-half the nominal annual rate. In case of quarterly compounding, the time factor will be four times the number of years, and the rate will be one-fourth the nominal.

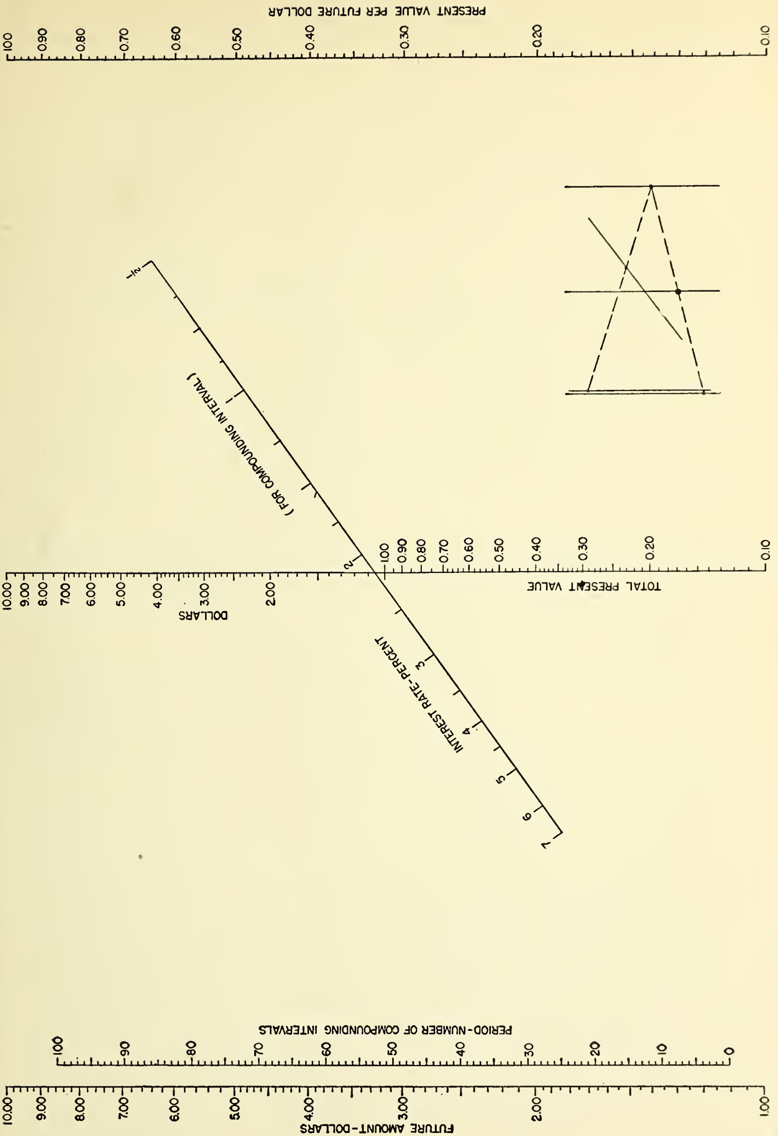
It is true that tables are more or less readily available for solving problems in this field, but this author has never been able to find tables which adequately covered all the conditions commonly met. Further, this chart covers fractional rates and gives more complete direct coverage of the time factor, reducing the need for interpolation. Then, too, when interpolation is necessary on the chart, it is simpler and quicker than interpolation by calculation of proportional differences.

**Procedure.** To use the chart, a straight line is projected through the scale points corresponding to time factor (*Period*) and *Interest Rate,* and the *Present Value per Future Dollar* is read where this line crosses the right-hand scale. A straight line from this point to the proper scaling on the extreme left-hand scale, *Future Amount,* will then intersect the center scale at a point giving the *Total Present Value.*

The *Future Amount* scale reads only from 1.00 to 10.00, but for larger quantities multiples of 10 should be used, the *Total Present Value* being adjusted in the same ratio as *Future Amount.*

Use of the chart is illustrated by determining the present value of an amount of $2,000, payable after 25 years, funds drawing interest at the rate of 4 per cent compounded semiannually. Twenty-five years will make the period 50 compounding intervals and the rate per compounding interval 2 per cent. A straight line through the scale point for 50 compounding intervals (*Period*) and 2 per cent (*Interest Rate*) intersects the scale for *Present Value per Future Dollar* at 0.373 (true value 0.3715).

To determine the full present value, this latter point is used with 2.00 on the *Future Amount* scale to establish the intersection on the *Total Present Value* scale at 0.745, and the actual total present value is 1,000 times this, or $745 (true value $743.60).


PRESENT VALUE PER FUTURE DOLLAR
TOTAL PRESENT VALUE
DOLLARS
INTEREST RATE-PERCENT
(FOR COMPOUNDING INTERVAL)
PERIOD-NUMBER OF COMPOUNDING INTERVALS
FUTURE AMOUNT-DOLLARS

# 21. Properties of Rectangles

**Method for the simultaneous solution** of several problems in the field indicated by the title. At a single reading can be had the area, the section modulus, the moment of inertia about the neutral axis, and the moment of inertia about an edge. At a single operation, solutions can be read for the mathematical expressions listed below, the terms $a$ and $b$ being taken as *Width* and *Height,* or depth, of the rectangle.

In each of the expressions stated below, the reference axis, for section modulus or moment of inertia, is taken as an axis parallel to the direction in which the term $a$ is measured.

1. Area of rectangle $= ab$
2. Section modulus $= ab^2/6$ (referred to neutral axis)
3. Moment of inertia $= ab^3/12$ (referred to neutral axis)
4. Moment of inertia $= ab^3/3$ (referred to one edge)

**Procedure.** To use the chart, a straightedge is laid to intersect at the given scaling values on the scales for *Width* and *Height.* At the intersection of this straight line with the other scales may be read the four properties as described.

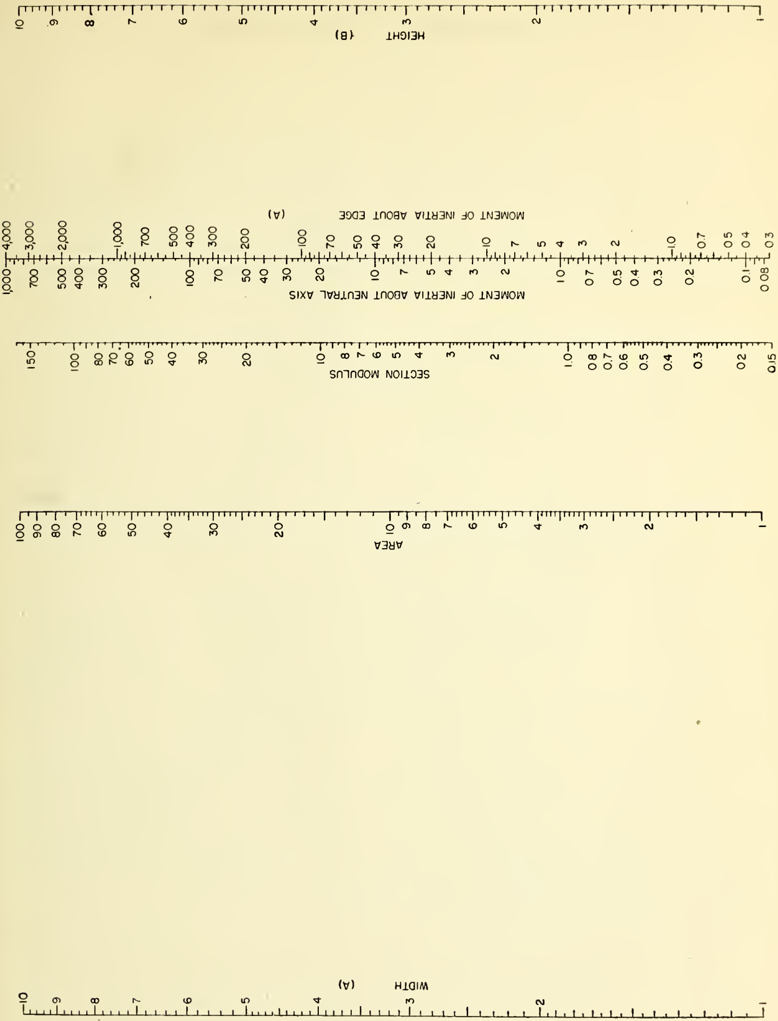
Should the given values lie beyond the scale limits, it is necessary only to make adjustments in the location of the decimal point. Thus, if the value for height lies between 10 and 100 (instead of being within the scale limits of 1 to 10), use the scaling point corresponding to one-tenth of the true value. The true results can then be obtained by multiplying the apparent results by proper factors, 10 for area, 100 for section modulus, and 1,000 for moment of inertia.

If the given value for height lies within the scaling limits but that for width does not, the operation would be started in the same way, but the same multiplier for apparent results would be used for all determinations, since they all vary directly with width.

To illustrate the use of the chart, the properties of a rectangle having a width of 8 units and a height of 2 units might be determined. A straight line cutting the *Width* scale at 8.0 and the *Height* scale at 2.0 intersects the other scales at points indicating the area to be 16 (correct value), the section modulus 5.3 (true value 5.33), moment of inertia about the neutral axis 5.35 (true value 5.33), and the moment of inertia about the lower edge 21.4 (true value 21.33).

If, instead of the given figures being as above, the width were 8 and the height 20, the same straight line would be established, and the same readings would be taken, but the results would be 160, 530, 5,350, and 21,400, respectively.

Should the figures fall outside the scaling in the opposite direction, the same general process is followed. Thus, for width of 4.0 and height of 0.8, the straight line is established as for 4.0 and 8.0, respectively, as the given figures, the apparent results being read as 32.0, 42.4, 172, and 685, respectively. Correction for the decimal point then gives the final results as 3.20, 0.424, 0.172, and 0.685, respectively, the numerically computed values actually being 3.20, 0.4267, 0.17067, and 0.68333, respectively.


HEIGHT (B)
MOMENT OF INERTIA ABOUT EDGE (A)
MOMENT OF INERTIA ABOUT NEUTRAL AXIS
SECTION MODULUS
AREA
WIDTH (A)

# 22. Area of Isosceles Trapezoids

**Method for the determination** of the areas of such figures, serving in the solution of problems involving excavation of trenches with sloping walls, enbankments with sloping sides, or flow area of water in canals. The mathematical expression for the area of such a figure is

$$\text{Area} = wh + h^2 \cot \phi$$

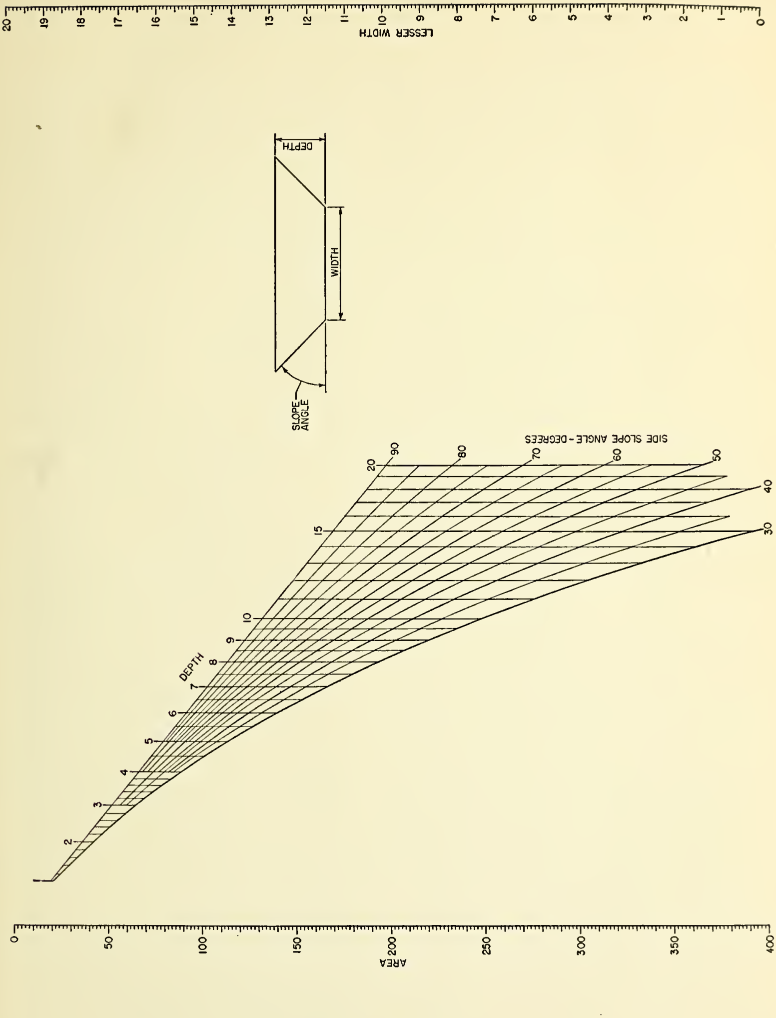
where $w$ is the width of the shorter of the two parallel sides of the figure, $h$ is the depth, or height, of the figure, and $\phi$ is the angle between the sloping side and the parallel sides.

**Procedure.** To use the chart, a straight line is established to intersect the scale points corresponding to three of the four variables, and the fourth figure is read at the intersection of the straight line with that scale. Thus, to determine the area of such a figure, where the shorter of the parallel sides has a length of 5 and the depth is 10, with an angle of 30° between sloping sides and parallel sides, a straight line is established by laying a straightedge from the scale point for 5 on the *Lesser Width* scale, through the vertical scale line for a *Depth* of 10 at its intersection with the sloping curve for 30° *Side Slope Angle.* The intercept of this line on the *Area* scale is at 223.2 (computed value 223.21).

Where given figures fall beyond the scale limits, or at points where accuracy is lost, the linear values may be multiplied by the same factor, one which serves to bring the dimensions into desired portions of the chart. The result from the chart has then only to be multiplied by the corresponding area factor to obtain the true result.

More specifically, if the linear dimensions are divided by a certain factor, to bring them into desired fields of the chart, then the area as read from the chart must be *multiplied* by the square of that factor. To illustrate this point, the area of a figure having a shorter side length of 24 and a depth of 20, with side slope angles of 60°, may be found. Instead of using the linear dimensions as given, they are divided by 2, *no change being made* as regards the angle. Then, the straight line is established from 12 on the *Lesser Width* scale through the intersection of the vertical line corresponding to a depth of 10 with the curve for 60° *Side Slope Angle.*

This line intersects the *Area* scale at a value of 177.5. This latter figure is then multiplied by 4 (the square of the 2 by which the original linear dimensions were divided) to give the area as 710 (true computed value 710.96).


LESSER WIDTH
DEPTH
WIDTH
SLOPE ANGLE
SIDE SLOPE ANGLE - DEGREES
DEPTH
AREA

# 23. Properties of Right Triangles

**Method for solving** various problems in this field. If a triangle, in addition to having one of its angles 90°, is defined as to the length of at least one side and the value of one of the other two angles or the length of one of the other two sides, all of the remaining values can be determined directly. It must be borne in mind, however, that the hypotenuse of the triangle, the side opposite the right angle, must always be evaluated on the chart by means of the family of curved lines. In general, better results with the other two side values will come of evaluating the greater of the two with respect to the vertical lines, using the scale at the bottom of the chart. Only one scaling is available for the acute angles, the sloping lines, but these give a measure of one angle, and the other is simply the difference between this and 90°.

**Procedure.** Interpolation between scale lines or curves will be necessary more often than not, but graphical interpolation is a simple matter and requires but little time and effort. If two side values are given, there are two alternative operations. If the two given sides are those enclosing the right angle, a point is located, by interpolation as necessary, corresponding to the scale values fixed by the vertical and horizontal lines. Interpolation between the nearest curves then gives directly the value for the length of the side opposite the right angle. A similar interpolation between the nearest sloping lines gives, similarly, the value for one of the acute angles.

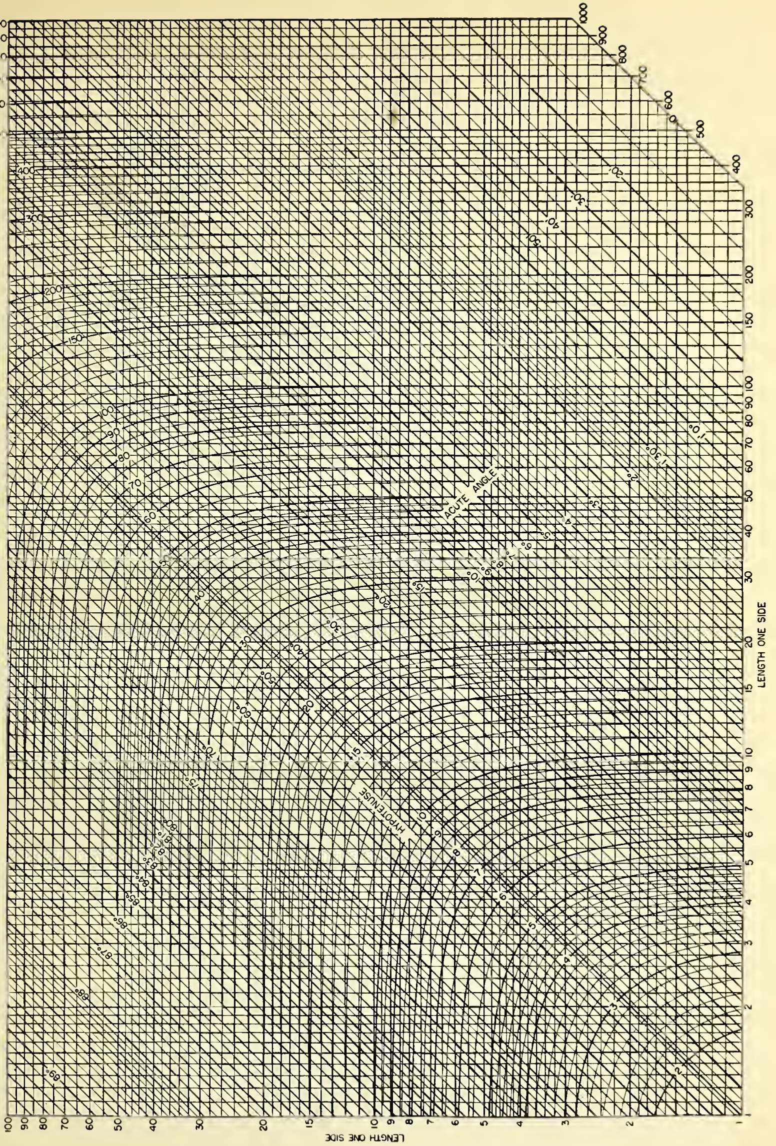
To illustrate such a problem, assume the lengths of the two sides enclosing the right angle as 150 and 65. The vertical line of 150 intersects the horizontal line of 65 between the curves for 160 and 170, interpolated as 163 (true value 163.48), and between sloping lines for 22° and 24°, interpolated as 23° 20′ (true value 23° 26′).

Where the given values are for one right-angle side and the hypotenuse, the point located would be that fixed by the hypotenuse-value curve intersecting either horizontal or vertical lines, the other values being read off, interpolating as necessary. Where hypotenuse length and one angle are known, the fixed point is determined by the intersection of the correct curve with the proper sloping line. Similarly, if one angle and the length of one of the right-angle sides are known, the fixed point is located by the intersection of the sloping line for the angle with either vertical or horizontal line for side length.

Although not sufficiently accurate for any but rough or preliminary solutions of problems, the chart has another use in saving time. This is in determining the more common trigonometric functions of angles, for they can be read directly from the chart. It covers sine, cosine, tangent, cotangent, secant, and cosecant. Procedure for each is as outlined below.

*Sine.* Locate the intersection of the sloping line of angle with the curve of 100, and read, interpolating between horizontal lines, on the left-hand scale *100 times* the sine of the angle.

*Cosine.* Locate the intersection of the


LENGTH ONE SIDE
ACUTE ANGLE
HYPOTENUSE
LENGTH ONE SIDE

sloping line of angle with the curve of 100, and read, interpolating between vertical lines, on the bottom scale *100 times* the cosine of the angle.

*Tangent.* Locate the intersection of the sloping line of angle with the vertical line of 100, and read, interpolating between horizontal lines, on the left-hand scale *100 times* the tangent of the angle.

*Cotangent.* Locate the intersection of the sloping line of angle with the horizontal line of *10,* and read, interpolating between vertical lines, on the bottom scale, *10 times* the cotangent of the angle.

*Secant.* Locate the intersection of the sloping line of angle with the vertical line of *10,* and read, interpolating between curves, *10 times* the secant of the angle.

*Cosecant.* Locate the intersection of the sloping line of angle with the horizontal line of *10,* and read, interpolating between curves, *10 times* the cosecant of the angle.

# Group III
# HYDRAULICS CHARTS

This group of 28 charts is gathered together into a single group as relating more or less directly to hydraulics and hydraulic equipment. However, it is limited in its coverage of the hydraulic field in that it relates to water almost entirely. The omission of coverage of fluids other than water was due to a feeling that such a coverage, to be adequate, would require a volume in itself. Then, too, this present volume is intended as a collection of charts of broader general engineering application.

The first four charts of this group give graphically, in the form of functional scales, means for converting various hydraulic units from one to another. In construction, they are similar to the charts in the first group in this volume, though in some there are multiple instead of single, but interrupted, scales. Each chart, naturally, carries its own explanatory notes.

Following these first charts there are a number having to do with water flow: over weirs of various types, through orifices, in open channels, and in pipes. In each case, again, the use of the chart is explained, and where proper sequence of operations is necessary, there is also a key diagram. In some of them, the mathematical basis may be subject to some question, but in each case, available experimental data have been compared and evaluated to arrive at basic data which practice has indicated to be sound.

There is a group of charts having to do with water hammer or surges in pipe lines, a subject not clearly understood by many engineers. It is for this reason that there is perhaps more space given to explanation than with charts having to do with more familiar subjects. These charts are based upon exact formulas, rather than any of the commonly used approximate methods, but the charts give solutions with little effort, thus achieving all of the advantages of those approximate methods while still having an exact basis.

Following the water-hammer charts, there are a number having general hydraulic use, dealing with pipe and tank volumes and stresses, as well as servomotor or hydraulic-ram determinations.

Next follows a series of six charts having to do with pumps and impulse- and reaction-type hydraulic turbines. These charts are intended not to be used as basis for design of such equipment but to give to the average engineer sufficient data for use in laying out preliminary plans for projects. Most of them represent graphically certain mathematical expressions which experience has shown to give average values for the figures sought. A final design may depart materially from such determinations, and each manufacturer may find consistent variations, but it is repeated that the charts are based upon general averages.

## Hydraulics Charts

Closing this group will be found two charts dealing with rainfall intensity and runoff. Again the charts have been based upon studies of experimental data from many sources and upon the use of various methods. And, again, the charts have been made to represent general average conditions but are certainly suitable for preliminary studies. In any precise investigation, the records and past performances in the vicinity in question must be the final basis for conclusions.

## Charts in Group III

24. Water Pressure, Unit Conversion
25. Water Quantity (Flow), Unit Conversion
26. Head and Velocity
27. Water Quantity (Static), Unit Conversion
28. V-notch Weir Discharge
29. Suppressed Weir Discharge
30. Contracted Weir Discharge
31. Submerged Weir Discharge
32. Orifice Discharge
33. Open-channel Flow
34. Pipe Flow and Loss
35. Pipe Flow Velocity
36. Pipe Flow Loss
37. Pressure Wave Velocity
38. Surge-pressure Rise
39. Maximum Surge-pressure Rise
40. Pipe or Tank Volume
41. Hydraulic-cylinder Force
42. Horizontal-tank Content
43. Cylindrical-tank and Pipe Strength
44. Jet Diameter and Theoretical Water Horsepower
45. Pump Size and Horsepower
46. Hydraulic-turbine Specific Speed
47. Hydraulic-turbine Speed
48. Hydraulic-turbine Inlet Diameter
49. Hydraulic-turbine Runner Diameter
50. Rainfall Intensity
51. Rainfall Runoff

# 24. Water Pressure, Unit Conversion

**Method for converting** water pressure measurements from and to any of the three common units for expressing such measurements. The basic relationships represented by the chart are

1 #/In.$^2$ = 2.307 Ft. $H_2O$ = 2.036 Ins. Hg

1 Ft. $H_2O$ = 0.8826 Ins. Hg = 0.4335 #/In.$^2$

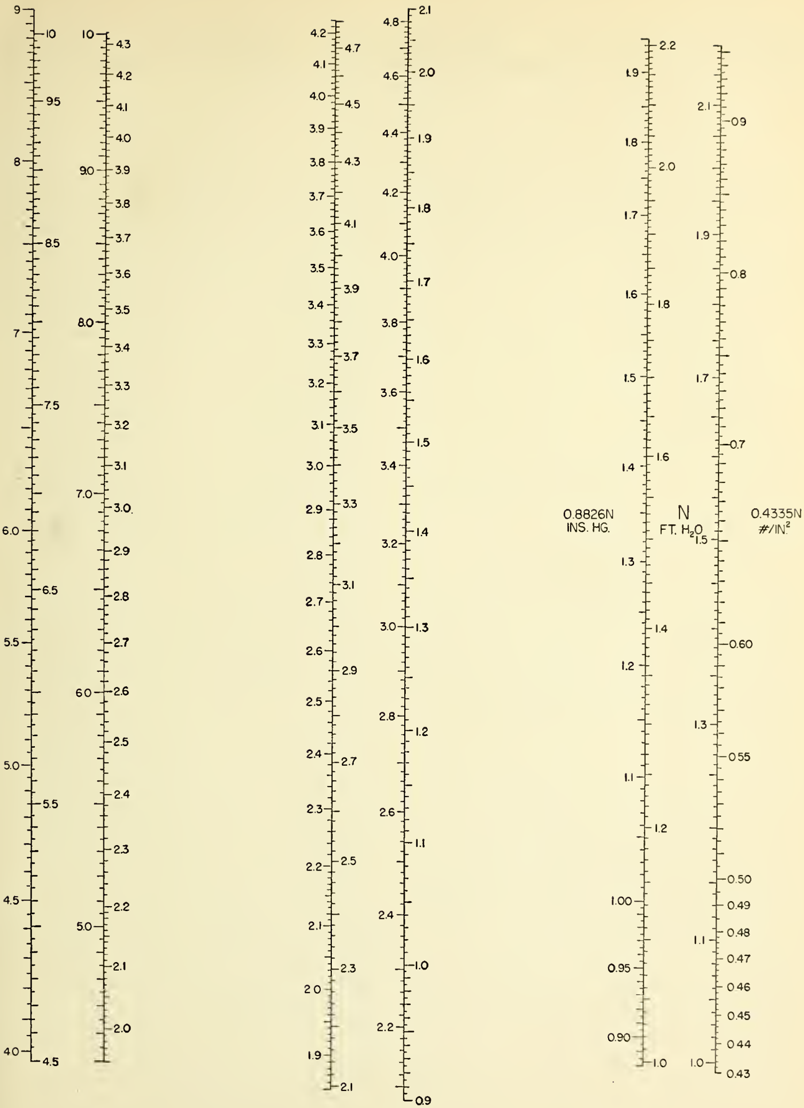
1 Ins. Hg = 0.491 #/In.$^2$ = 1.133 Ft. $H_2O$

where #/In.$^2$ = pounds per square inch, Ft. $H_2O$ = feet of water, and Ins. Hg = inches of mercury.

It is true that tables are more or less readily available for making such conversions, but even so interpolation, necessary more often than not, is simpler graphically than arithmetically. Further, this chart gives in a single sheet a complete coverage of all of these conversions.

**Procedure.** The chart consists of two interrupted scales, double-scaled. The inner scaling on each is in *Feet of Water,* with scaling for *Inches of Mercury* on the outer side of one and *Pounds per Square Inch* on the other. To convert from or to *Feet of Water* to or from either of the other units, it is necessary only to read opposite scalings on the same line. However, the conversion between *Pounds per Square Inch* and *Inches of Mercury* requires the intermediate step of conversion to *Feet of Water* and transfer to the other scale.

For instance, to convert 1.5 pounds per square inch, the figure directly opposite this scaling appears as 3.46 feet of water (true value 3.465), and opposite the same scaling on the other line is read 3.055 inches of mercury (true value 3.054). For values beyond the chart scaling, all scales are in direct proportion, so it is necessary only to move the decimal point. Thus in the range of 10 to 100 feet of water, the ranges will be approximately 9 to 90 inches of mercury and 4.3 to 43 pounds per square inch.


0.8826N
INS. HG.
N
FT. H2O
0.4335N
#/IN²

# 25. Water Quantity (Flow), Unit Conversion

**Method for converting** water quantity measurements (of flow) from and to any of the four common units for expressing such measurements. The basic relationships represented by the chart are

1 Ac.-Ft./Day = 0.504 $Ft.^3$/Sec.

= 226.3 Gal./Min. = 0.326 Mgd.

1 $Ft.^3$/Sec. = 448.8 Gal./Min.

= 0.646 Mgd. = 1.983 Ac.-Ft./Day

1 Gal./Min. = 0.00223 $Ft.^3$/Sec.

= 0.00144 Mgd. = 0.00442 Ac.-Ft./Day

1 Mgd. = 3.07 Ac.-Ft./Day

= 1.547 $Ft.^3$/Sec. = 694.4 Gal./Min.

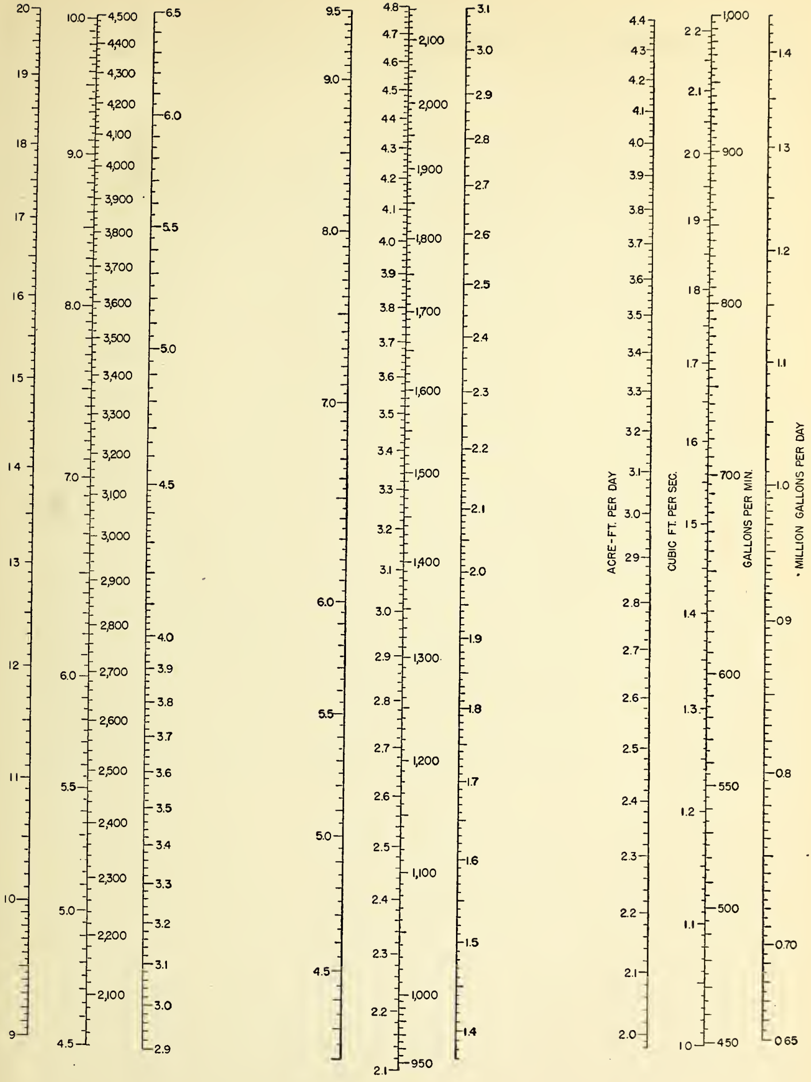
where Ac.-Ft./Day (Acre-Ft. per Day on chart) = acre-feet per day (24 hours), $Ft.^3$/Sec. (Cubic Ft. per Sec. on chart) = cubic feet per second, Gal./Min. (Gallons per Min. on chart) = gallons per minute, and Mgd. (Million Gallons per Day on chart) = millions of gallons per day (24 hours).

It is true that tables are more or less readily available for making such conversions, but even so, interpolation, necessary more often than not, is simpler graphically than arithmetically. Further, this chart gives in a single sheet a complete coverage of all of these conversions.

**Procedure.** The chart consists of three interrupted (three-section) scales, of which one is scaled on opposite sides for two of the measurement units, and each of the others is scaled for an additional one. In any case, a straight line, intersecting the three lines (four scales) at right angles with the lines, will intersect those lines at scale points which indicate the true related values in terms of the four units. For making determinations or conversions, an ordinary drafting triangle will give such a crossline, but a more convenient method is to use a xylonite or other transparent sheet with right-angled scratched lines. Such an instrument will also aid in the use of other charts for conversions included in this present group.

To illustrate the use of the chart, the equivalent quantities may be determined for 1.5 cubic feet per second. The scaling directly opposite this scale point gives one equivalent as 673 gallons per minute (true value 673.2). A line through this scaling perpendicular to the scale lines intersects one of the other lines at a scale value of 0.97 million gallons per day (true value 0.969), and the other at scaling 2.975 acre-feet per day (true value 2.9745).

For values beyond the primary scalings, it need only be remembered that all scales are in direct proportion. Thus if one value used is ten times the primary scale value, all of the other measurements will be ten times the scale figure.


ACRE-FT. PER DAY
CUBIC FT. PER SEC.
GALLONS PER MIN.
• MILLION GALLONS PER DAY

# 26. Head and Velocity

**Method for solving** problems having to do with the relationship between head and velocity in water flow. The chart serves to determine either the velocity head due to a known flow velocity or the theoretical velocity of a jet discharged under a known head. The basic relationship is expressed by the two equations

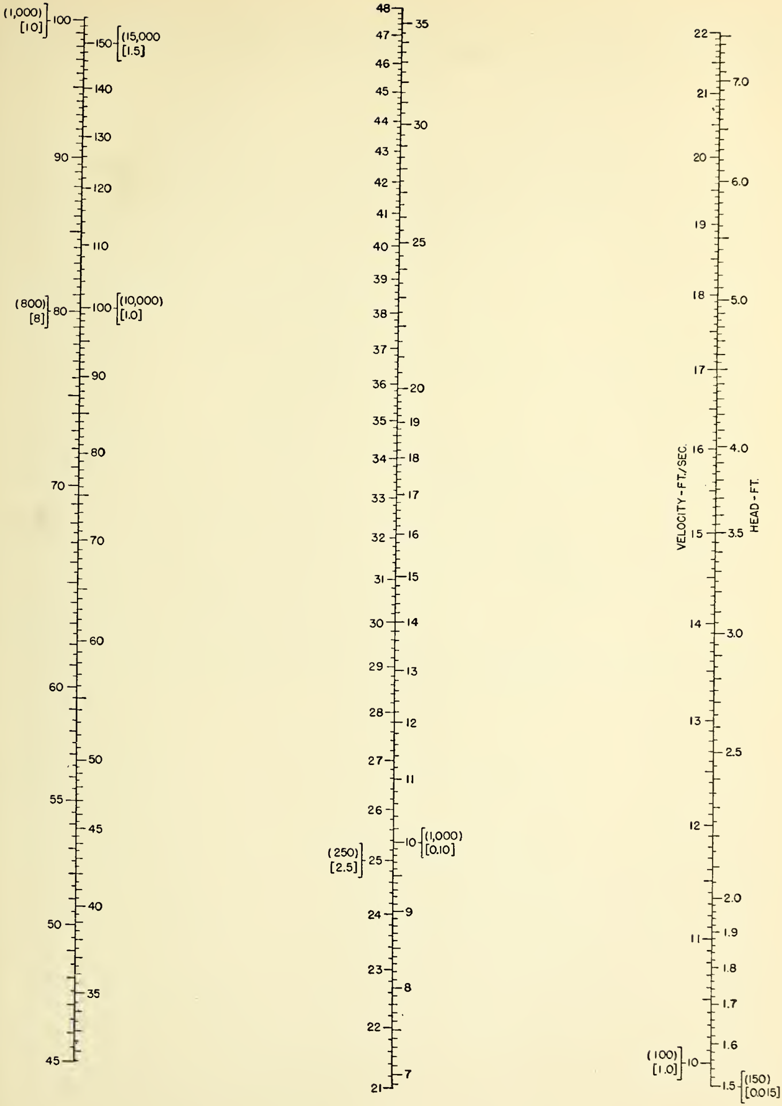
$$H = V^2/2g \qquad \text{and} \qquad V = \sqrt{2gH}$$

where $H$ is head in feet, $V$ is velocity in feet per second, and $g$ is gravitational acceleration, taken as 32.162.

It is true that tables are more or less readily available for making such determinations, but all that have come to the attention of this writer have been limited in scope, with tabulation carried to but two significant figures. Such tables, even within their range, involve interpolation, for any accurate work, in almost every case, and while interpolation may be necessary almost as often with this chart, this process is much simpler and easier when done graphically than when done arithmetically.

**Procedure.** In using the chart, the scale point corresponding to the given value is located on the proper scale, interpolating as necessary. Then, the scaling on the opposite side of the line at the same point gives the desired value. Thus, to determine the theoretical spouting velocity of a jet under a head of 48 feet, the scale point for this figure is located on the *Head, Feet* scale. The scaling on the opposite side of the line indicates *Velocity* in feet per second as 55.6 (true value 55.56).

Secondary scalings are indicated to show results where location of the decimal point deviates from that of the primary scaling. Thus, to find the velocity head resulting from a flow of 3.6 feet per second (in the range from 1.0 to 10), the scale point for *36* of the primary scaling is located on the *Velocity, Feet per Second* scale. The scaling on the opposite side of the line indicates *Head, Feet* as 20.2. But the secondary scaling indicates that the true value falls between 0.1 and 1.0, so the result is a velocity head of 0.202 foot (true value 0.2015).


(1,000) [10] 100
150 (15,000 [1.5]
(800) [8] 80
100 (10,000) [1.0]
(250) [2.5] 25
10 (1,000) [0.10]
(100) [1.0] 10
1.5 (150) [0.015]
VELOCITY - FT./SEC.
HEAD - FT.

# 27. Water Quantity (Static), Unit Conversion

**Method for converting** water quantity measurements from and to any of the three common units for expressing such measurements. The basic relationships represented by the chart are

1 Cubic Foot = 7.4802 Gallons

= 62.428 Pounds

1 Gallon = 8.33 Pounds = 0.1337 Cubic Foot

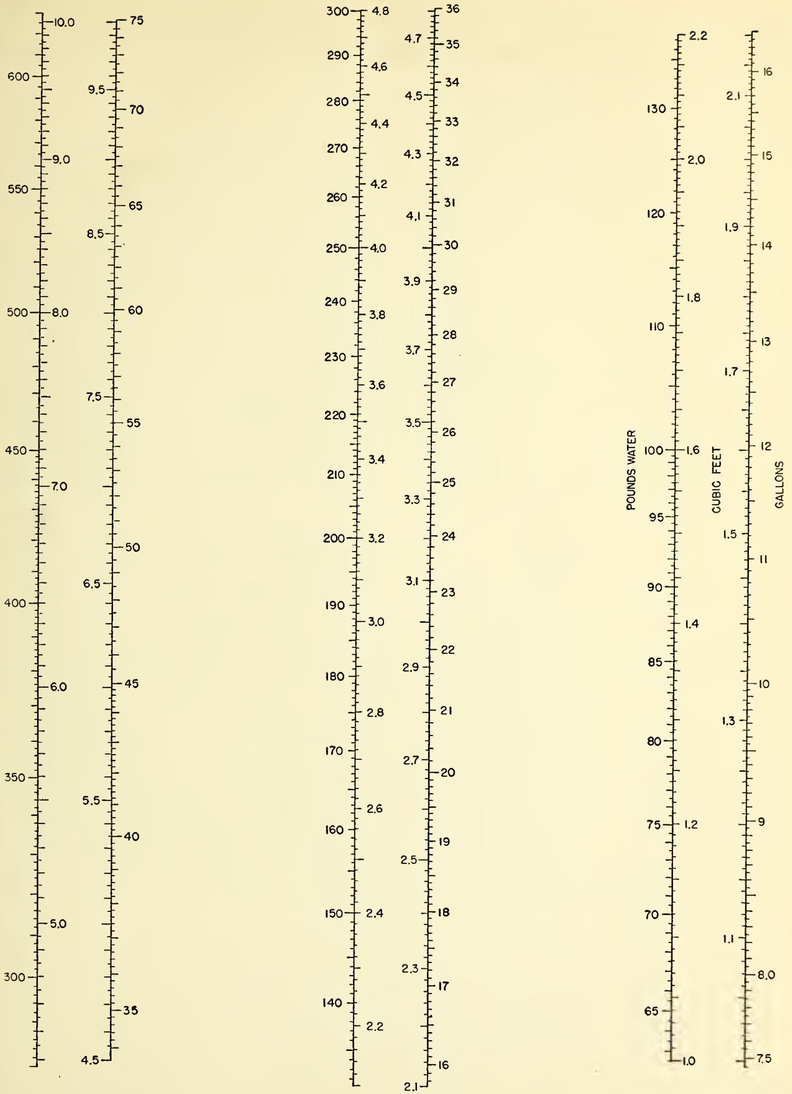
1 Pound = 0.01602 Cubic Foot

= 0.1200 Gallon

It is true that tables are more or less readily available for making such conversions, but even so, interpolation, necessary more often than not, is simpler graphically than arithmetically. Further, this chart gives in a single sheet a complete coverage of all of these conversions.

**Procedure.** The chart consists of two interrupted scales, each double-scaled. The inner scaling on each is in *Cubic Feet,* with scaling for *Gallons* on the outer side of one and *Pounds* on the other. To convert from or to *Cubic Feet* to or from either of the other units, it is necessary only to read opposite scalings on the same line. However, the conversion between *Gallons* and *Pounds* requires the intermediate step of conversion to *Cubic Feet* and transfer to the other scaled line.

For instance, to convert 50 gallons, the figure directly opposite this scaling appears as 6.68 cubic feet (true value 6.685), and opposite the same scaling on the other line is read 418 pounds (true value 416.5). For values beyond the chart scaling, all scales are in direct proportion, so it is necessary only to move the decimal point. Thus in the range of 10 to 100 cubic feet, the ranges will be approximately 75 to 750 for gallons and 625 to 6,250 for pounds.


POUNDS WATER
CUBIC FEET
GALLONS

# 28. V-notch Weir Discharge

**Method for the determination** of the flow over triangular, or V-notch, weirs of various angles and under the normal range of head of water. It is true that tables are available for notches of certain angles, but this writer has seen none for angles other than 90° and 60°. But even for these angles, the chart is convenient, in that any interpolation is graphical rather than arithmetical. For other angles, the chart will save enormously, in avoiding reference to logarithmic (or exponential) and trigonometric tables, as well as a multiplication operation.

The basic formula for the flow of water over a V notch is

$$Q = \tfrac{4}{15} c\sqrt{2g} L H^{1.5}$$

where $Q$ is discharge in cubic feet per second, $c$ is an experimental coefficient, taken as 0.59 for this chart, $g$ is gravitational acceleration (32.162 feet per second per second), $L$ is the length of the theoretical crest of the water stream through the notch, in feet, and $H$ is the head on the notch, measured in feet above the point (lower apex) of the notch, to the water-level elevation at a point sufficiently upstream to avoid the downdrop curve. ($L$ is measured at this elevation.)

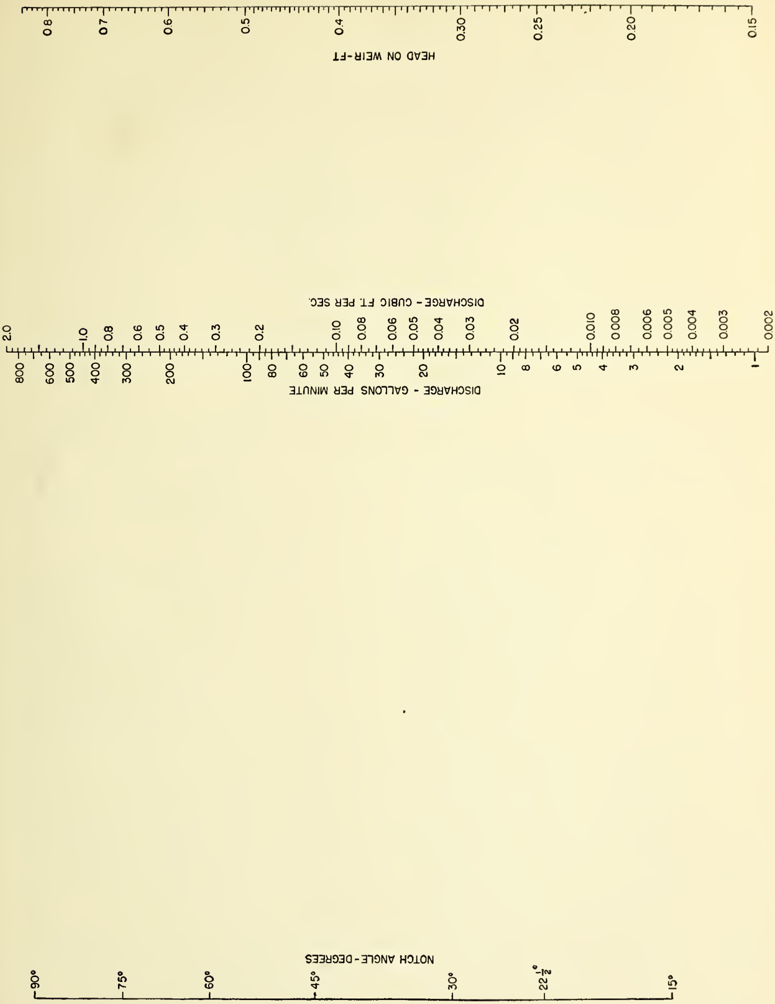
Since $L$ is a direct function of the head and notch angle, the equation can be simplified to

$$Q = 2.53H^{2.5} \tan (\theta/2)$$

where $\theta$ is the angle between the sides of the V notch. Naturally, the layout of the notch is such that it is symmetrical with respect to a vertical line.

The use of a value of 0.59 for $c$ might be questioned, since various authorities place its value variously within the range from 0.57 to 0.60. The bulk of available data, however, indicates the chosen figure as a reasonable average of experimental results.

**Procedure.** To use the chart, a straight line is projected from the known value of the notch angle to the scale value of the observed *Head on Weir,* and at the intersection of this line with the central scale is read the *Discharge,* in gallons per minute or cubic feet per second. For instance, the flow over a 90° notch under a head of 9 inches (0.75 foot) is determined by a straight line from 90° on the *Notch Angle* scale to 0.75 on the *Head on Weir* scale, which intersects the *Discharge* scale at a scaling of 545 gallons per minute. Tabulations of experimental data give actual results from 533 to 550.


HEAD ON WEIR-FT
DISCHARGE - CUBIC FT. PER SEC.
DISCHARGE - GALLONS PER MINUTE
NOTCH ANGLE - DEGREES

# 29. Suppressed Weir Discharge

**Method for the determination** of the flow of water over rectangular weirs where there are no end contractions. The formula used in setting up the chart is that due to Rehbock, which is widely used where maximum accuracy is desired. But it is complex and time-consuming, shortcomings that are avoided by use of the chart. True, tabulations are more or less readily available to cover the normal range, but these involve interpolation more often than not, and even where interpolation is necessary with the chart, it is a simpler operation than with tables.

According to Rehbock, the discharge over a weir of this type, in cubic feet per second, is

$$Q = L[3.228 + 0.435(H/P)]H^{1.5}$$

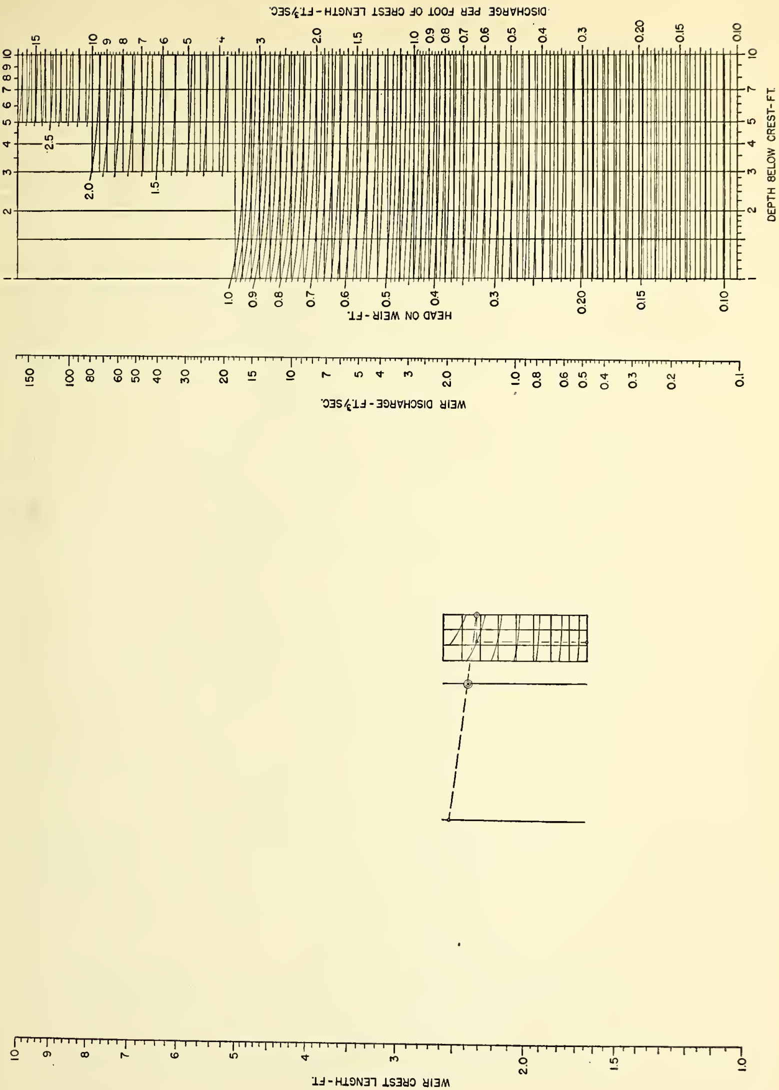
where $L$ is the length of the weir crest (= approach-channel width) in feet, $P$ is the height of the weir crest above the bottom of the approach channel in feet, and $H$ is a quantity equal to the head in feet above the crest plus 0.0036.

At the right-hand side of the chart there is an intersection-type nomograph which deals with the quantities other than crest length. However, the scale readings for head are true observed head above the crest, the added constant being taken care of in setting up the chart. Values of head are represented by the family of curves, with numerical values marking their extreme left ends. Values for depth of channel below weir crest are represented by vertical lines according to the scale at the bottom. A scaling at the right edge of this section represents the discharge per foot of crest length.

**Procedure.** To use the chart, the point of intersection is located in the right-hand section just described, which corresponds to the observed values of *Head on Weir* and *Depth below Crest,* interpolating as may be necessary. Then follow horizontally across this section of the chart to its right-hand edge, and there may be read the discharge per foot of crest length. Then, a straight line through this point and the scale point corresponding to *Weir Crest Length* on the scale at the left side of the chart will intersect the *Weir Discharge* scale at a scale point corresponding to the total discharge.

For instance, assume an approach channel 6.0 feet wide (crest length) and 4.0 feet deep below the crest, with water overflowing with a measured head above the crest of 0.5 foot. Locate the intersection point between the curve for 0.5-foot head and the vertical line for 4.0-foot depth; then follow horizontally to the scale at the right-hand side of this section. There the discharge per foot of crest length is read as 1.17 cubic feet per second. A straight line through this point and the scale point for 6.0-foot crest length intersects the middle scale at a point showing the total discharge to be 7.00 cubic feet per second.

Where the crest-length value is outside the scaling limits. a scale point for 0.1 or 10.0 times the actual length should be used on the chart, the *Discharge* determination afterward being multiplied by 10.0 or 0.1, respectively.


DISCHARGE PER FOOT OF CREST LENGTH - FT.³/SEC.
DEPTH BELOW CREST-FT.
HEAD ON WEIR - FT.
WEIR DISCHARGE - FT.³/SEC.
WEIR CREST LENGTH - FT.

# 30. Contracted Weir Discharge

**Method for the determination** of the flow of water over rectangular weirs with full end contractions. The formula used in setting up the chart is not one of those in common use, but it has the advantage of retaining a high degree of accuracy, or rather, of conformance with experimental results, over the full range from heads down to a hundredth of the crest length up to those of twice the crest length.

It is true that tabulations are available which give discharges of this type of weir, some based upon experimental data, some on computation by use of various formulas, and some mixtures of the two. But in any case, interpolation is often necessary and is much more involved and time-consuming than interpolation graphically by use of the chart. When tables are not available or do not provide adequate coverage, arithmetical computation is a laborious process, even with the simplest of the formulas. On the other hand, this chart should cover the entire practical range of the variables involved in such measurements.

The equation upon which the chart is based is not, as was stated, in common use, though its addition to the Power Test Code was proposed some years ago. It is

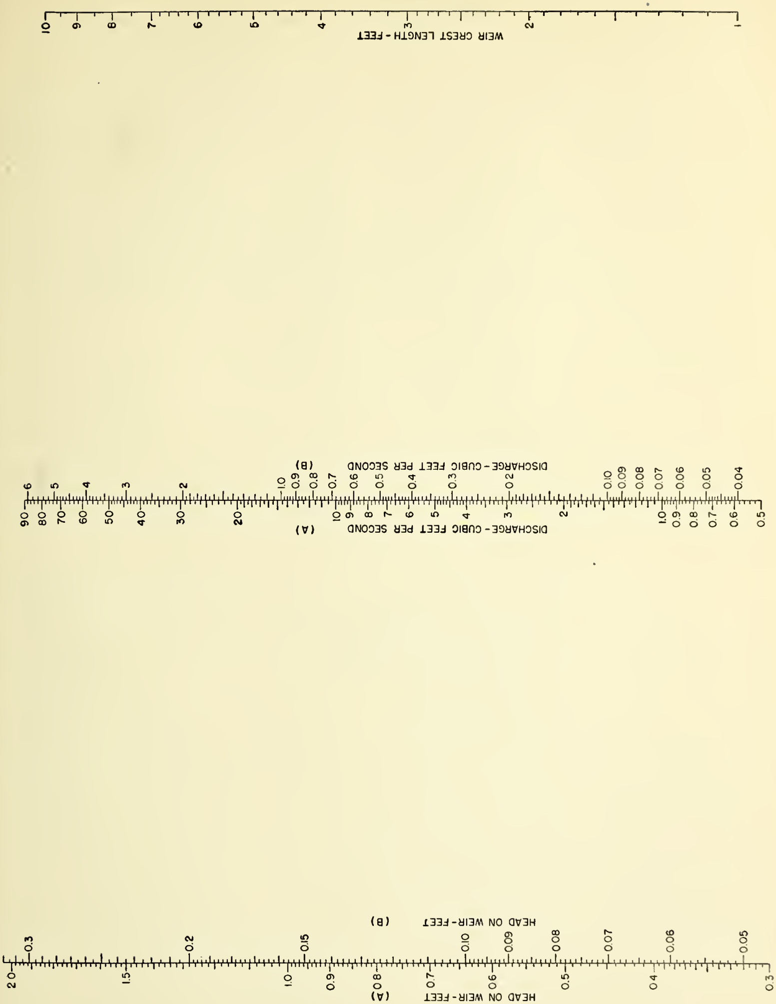
$$Q = 3.125LH^{1\frac{7}{16}}$$

where $Q$ is discharge in cubic feet per second, $L$ is the crest length in feet, and $H$ is the effective head above the crest, in feet, including velocity head at the point of measuring actual elevation above the crest. The exponent $1\frac{7}{16}$ appears to take care of the variation of the coefficient with different ratios of head to crest length, even to the extremes mentioned above.

**Procedure.** To use the chart, a straight line is projected from the known value of *Crest Length* through the observed value of *Head on Weir,* on their respective scales, and the *Discharge* is read on that scale at the line's intersection. On the *Head on Weir* axis there are two scalings, *A* and *B,* and likewise there are two, *A* and *B,* on the *Discharge* axis. In each case, the scalings of like designations should be used, *A* with *A* and *B* with *B*. Where the crest length is outside the scaling limits, a scale point for 0.1 or 10 times the actual length should be used, afterward multiplying the *Discharge* result by 10 or 0.1, respectively.

By way of illustration, we may determine the discharge over a 3-foot weir under a head of 1.0 foot. A line through the scale point for 3.00 on the *Weir Crest Length* scale and that for 1.00 on the *Head on Weir A* scale intersects the *Discharge A* scale at 9.30 cubic feet per second. If the crest length were 0.3 foot, the discharge would be 0.93, and with the length 30.0 feet, it would be 93.0. With 3-foot crest and 0.1-foot head, the *B* scalings would give a discharge of 0.333 cubic foot per second.

Where the crest-length value is outside the scaling limits, a scale point for 0.1 or 10.0 times the actual length should be used on the chart, the *Discharge* determination afterward being multiplied by 10.0 or 0.1, respectively.


WEIR CREST LENGTH - FEET
DISCHARGE - CUBIC FEET PER SECOND (B)
DISCHARGE - CUBIC FEET PER SECOND (A)
HEAD ON WEIR - FEET (B)
HEAD ON WEIR - FEET (A)

# 31. Submerged Weir Discharge

**Method for the determination** of the flow of water over rectangular submerged weirs without end contractions. The formula used in setting up the chart is that due to Herschel, which is widely used. But its use is time-consuming, a shortcoming avoided by use of this chart. Tabulations may be available setting forth discharges under various conditions, but none has come to the attention of this author. But even if they are available, any interpolation, necessary more often than not, would involve arithmetical computation of proportional parts, whereas interpolation on the chart, being visual, is far easier.

According to Herschel, the discharge over a weir of this type, in cubic feet per second, is

$$Q = 3.33L(nH)^{3/2}$$

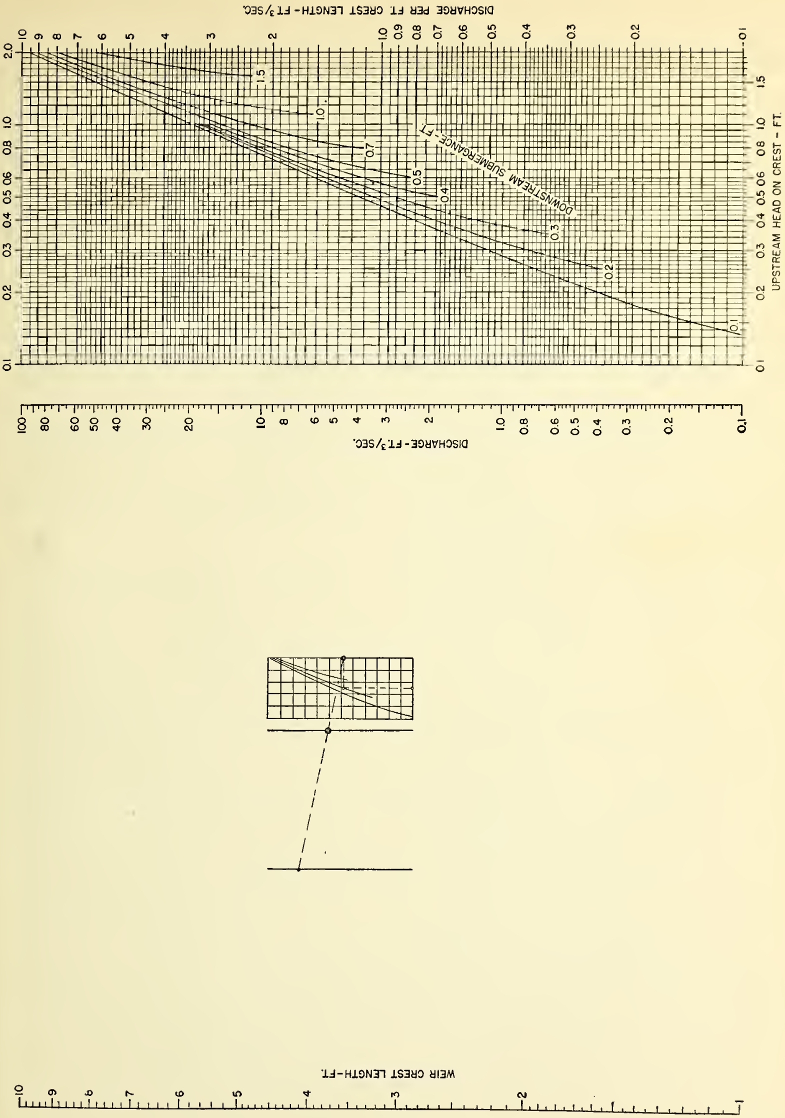
where $L$ is the length of the weir crest (= approach-channel width) in feet, $H$ is the upstream head above the weir crest, also in feet, and $n$ is an experimental factor varying with the ratio between downstream and upstream heads or heights of water level above the elevation of the weir crest.

At the right-hand side of the chart there is an intersection-type nomograph which deals with the quantities other than crest length. Here, values of *Downstream Submergence,* or head, are represented by the family of curves, while values of *Upstream Head on Crest* are represented by the vertical lines. The constant, the experimental factor, and the exponent are all brought into the operation by which the intersection of any curve with any vertical line can be related to the horizontal lines to obtain the value of *Discharge per Foot Crest Length.*

**Procedure.** To use the chart, the point of intersection corresponding to observed values of upstream and downstream head is located in the right-hand section just described, interpolating as necessary. Then follow horizontally across this section to the right-hand edge, and there may be read *Discharge per Foot Crest Length.* Then a straight line through this point and the scale point corresponding to *Weir Crest Length* on the scale at the left-hand side of the chart, will intersect the *Discharge* scale at a point corresponding to the total discharge.

For instance, assume an approach-channel width and crest length of 7.00 feet, with upstream and downstream heads 1.50 and 1.00 feet, respectively. Locate the intersection point of the curve of 1.00 *Downstream Submergence* with the vertical line of 1.50 *Upstream Head on Crest,* and then follow horizontally to the scale at the right-hand side of this section. There the *Discharge per Foot Crest Length* is read as 4.45 cubic feet per second. A straight line through this point and the scale point for 7.00 *Weir Crest Length* intersects the *Discharge* scale at 31.3 cubic feet per second.

Where the crest length is outside the scaling limits, a scale point for 0.1 or 10.0 times the actual length should be used, the *Discharge* determination afterward being multiplied by 10.0 or 0.1, respectively.


DISCHARGE PER FT. CREST LENGTH - FT.³/SEC.
UPSTREAM HEAD ON CREST - FT.
DOWNSTREAM SUBMERGANCE - FT.
DISCHARGE - FT.³/SEC.
WEIR CREST LENGTH-FT.

# 32. Orifice Discharge

**Method for the determination** of the discharge of water through orifices of various shapes and sizes and under various heads. It is true that there are available tabulations for discharges of round orifices under the assumption of perfect flow, and some tables for other conditions have come to this author's attention. However, interpolation is necessary more often than not, and there is the further requirement, in most cases, of applying a coefficient to correct for the form of orifice. The chart, however, makes any necessary interpolation a visual matter, always easier than arithmetical, and it offers direct adjustment for orifice form.

The formula upon which the chart is based is the conventional one for orifice discharge, which, in cubic feet per second, is

$$Q = CA\sqrt{2gH}$$

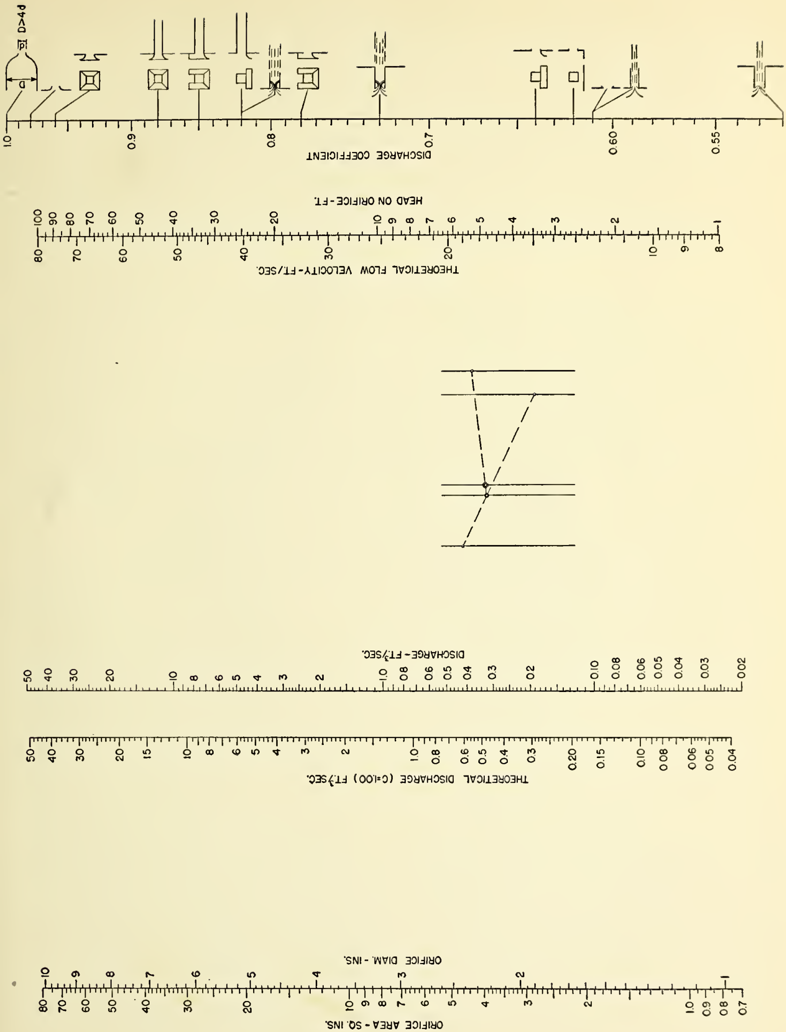
where $A$ is the orifice area in square feet, $g$ is gravitational acceleration in feet per second per second, $H$ is the effective head of water on the orifice center line, in feet, and $C$ is a coefficient of discharge. This last term is equal to 1.00 for perfect discharge without loss and with various forms of entrances and shapes of orifices will have lower values.

One of the chart scales covers the usual range of this coefficient, but on that same scale diagrammatic illustrations of various forms of orifices are shown, with indicating lines to the scale point, in each case, which has been selected as a fair average for its coefficient. Both round and square orifices are illustrated, with different forms of entrance and outlet. For rectangular orifices departing not too far from square, the coefficient indicated is accurate enough for general purposes, but where greater accuracy is required, recourse should be had to tables which are available, and the coefficient so found should be used on the chart.

In representing on the chart the various types of orifices, it is hoped that the figures identify themselves. However, in case this objective has not been realized, they are more specifically identified below, together with the assumed values of coefficients, which appear at the end of the line in the tabulation on page 78.

**Procedure.** The use of the chart is illustrated in the key diagram, and after initial use, that should suffice as a guide in procedure. But, more specifically, the first step is to project a straight line through the scale points corresponding to measured values of *Head on Orifice* (center) and *Orifice Diameter* or (on the opposite side of the same scale) *Orifice Area*. The scale value at the intersection of this line with the *Theoretical Discharge* scale gives the discharge, in cubic feet per second, with a coefficient of discharge of 1.00.

A straight line established by this point and the scale point corresponding to the proper coefficient of discharge, or the index point identified by the form symbol, will then intersect the *Discharge Quantity* scale at a scale point that gives the actual discharge. Where the actual values of head are beyond the scale limits of the chart, the true value may be multiplied by 0.01


D>4d
DISCHARGE COEFFICIENT
HEAD ON ORIFICE - FT.
THEORETICAL FLOW VELOCITY - FT./SEC.
DISCHARGE - FT.³/SEC.
THEORETICAL DISCHARGE (C=1.00) FT.³/SEC.
ORIFICE DIAM. - INS.
ORIFICE AREA - SQ. INS.

or 100 to arrive at a value within the limits, making the change in decimal point of the *Quantity* value as indicated by the auxiliary scalings appearing at the ends of the scales. A similar adjustment may be made for orifice area, bearing in mind that the discharge varies directly with the area; *i.e.,* for an area 10.0 times the scale value, the discharge will be 10.0 times that indicated, but for a *diameter* 10.0 times a scale value, the discharge will be 100 times that indicated.

To illustrate by determining the discharge through a sharp-edged round orifice 2.00 inches in diameter under a head of 75.0 feet, a line through 75.0 feet head and 2.00-inch diameter intersects the *Theoretical Discharge* axis at a scale value of 1.50 cubic feet per second. A line through this point, in turn, and the index point for round, sharp-edged orifice (coefficient = 0.61) intersects the *Discharge Quantity* scale at a value of 0.92 cubic foot per second.

| | |
|---|---|
| 1. Well-designed, bell-mouthed orifice, with entrance channel having an area at least 16 times that of the orifice | 1.00 |
| 2. Bell-mouthed orifice in wall of water container | 0.98 |
| 3. Square orifice, with (elliptically) curved entrance guide surfaces all around opening | 0.96 |
| 4. Same as (3), but with discharge through confined channel of same size and shape in cross section as orifice and of length of three or more times the shortest dimension of the orifice itself | 0.88 |
| 5. Same as (4), but with curved entrance guide surface omitted on top side of orifice | 0.85 |
| 6. Same as (4), but with curved entrance guide surface along bottom edge of orifice only | 0.82 |
| 7. Round orifice with tube of same diameter, flowing full, entrance flush with inner surface of water container | 0.82 |
| 8. Same as (3), but with curved entrance guide surface omitted on top side of orifice | 0.78 |
| 9. Orifice being sharp-edged end of tube extending into water container, round, running full | 0.73 |
| 10. Same as (3), but with curved entrance guide surface along bottom edge of orifice only | 0.64 |
| 11. Square, sharp-edged orifice | 0.62 |
| 12. Round, sharp-edged orifice | 0.61 |
| 13. Same as (7), but tube not running full | 0.61 |
| 14. Same as (9), but tube not running full | 0.52 |

# 33. Open-channel Flow

**Method for the solution** of problems having to do with the flow of water in open channels, canals, flumes, and ditches. There are several formulas in general use for such problems and any number of charts for simplification of their use. But, though there are other charts available, this one is offered as having two slight advantages. The first is that it is simpler to use than most of those available, and the second is that it covers a wider range of the variables than most.

The basis upon which the chart was designed is the Manning formula, wherein the flow velocity, in feet per second, is

$$V = 1.486R^{2/3}S^{1/2}/n$$

where $R$ is the hydraulic radius of the flow section, in feet, $S$ is the slope of the flow line, expressed as a decimal value of drop per foot of distance, and $n$ is a roughness coefficient. The hydraulic radius may be further explained as being the result obtained by dividing the area of the stream cross section by the length, measured transversely on that section, of the wetted perimeter of the section.

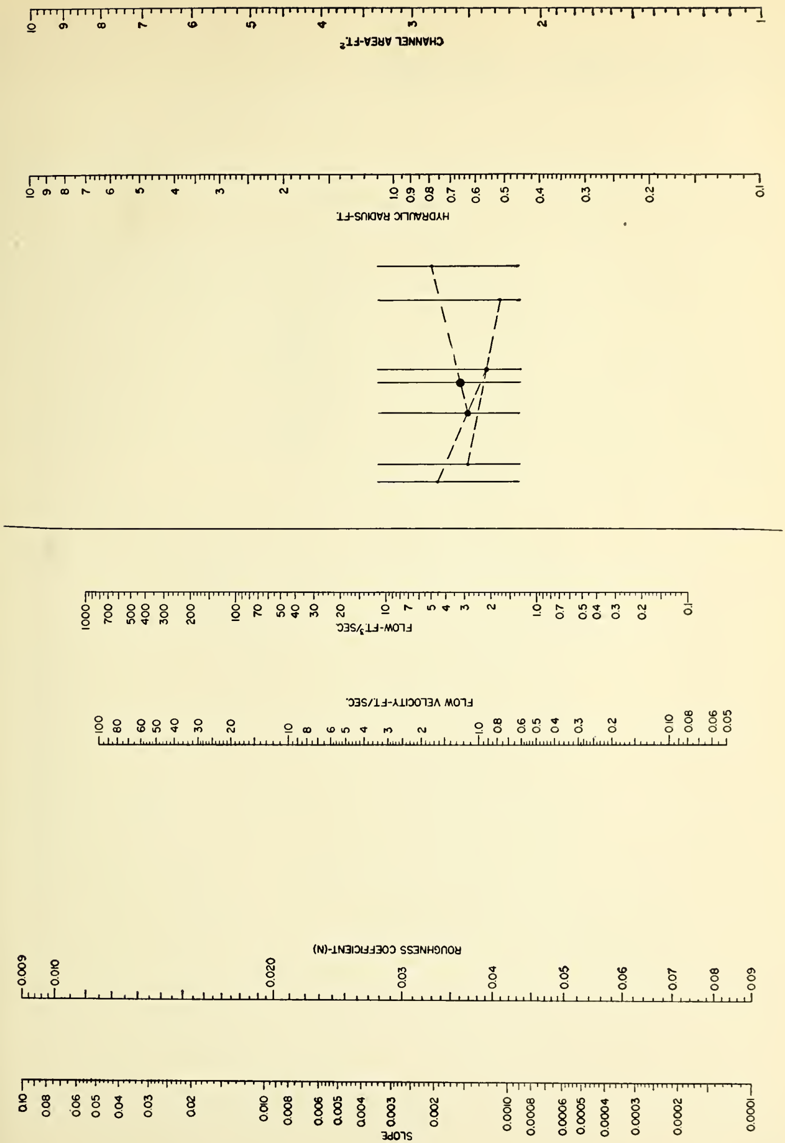
The roughness coefficient $n$ is the same as that used in the more complex Kutter formula. It varies from a minimum value of about 0.009, for extremely smooth surfaces, such as when enameled, to about 0.150 for channels of rough surfaces, with irregular section and with timber, boulders, or other obstructions in the flow section. However, it is seldom that values above 0.075 will be encountered in problems of controlled flow. As a matter of fact, only in studies of flood flow will much higher values be encountered, and even then they will very rarely exceed 0.100.

**Procedure.** To use the chart to determine the flow in a channel under known conditions, the key diagram on the chart indicates the sequence of operations. First, a straight line is established, passing through the scale points corresponding to the known values of *Roughness Coefficient* and *Hydraulic Radius* on those respective scales. This line will intersect the blank axis, which, though not scaled, fixes a point that characterizes the channel. A second line established by this point and the proper scale value on the *Slope* scale will then intersect the *Flow Velocity* scale at a scale point that gives the flow velocity in feet per second. Then, a third line established by this point and the scale value corresponding to the area of the flow cross section will intersect the *Flow* axis at a scale point which establishes the total flow in cubic feet per second.

Where the cross-section area is such as to fall outside the scale limits, a value of 0.1 or 10.0 times the true area is used on the chart, and the final flow figure is then multiplied by 10.0 or 0.1, respectively, as the case may be, to obtain the true value. In using the chart, it is essential that the intersections be established by points on the axis groups as explained. However, the procedure may be reversed or started from both ends to establish a factor intermediate, such as slope.

For instance, it is often the practice in preliminary design work on channels to start with a limiting figure for velocity, established by the material through which the channel passes, and this, with the known requirement for total flow, determines the cross-section area and the hydraulic radius. A line established by this latter figure and the assumed roughness coefficient then gives an intercept on the blank axis, and this point and the assumed flow velocity define the line which establishes the required slope. Or if the slope is limited, the slope and velocity establish an intercept on the blank axis, which by trial and error must be reconciled with the relation between channel area and hydraulic radius.

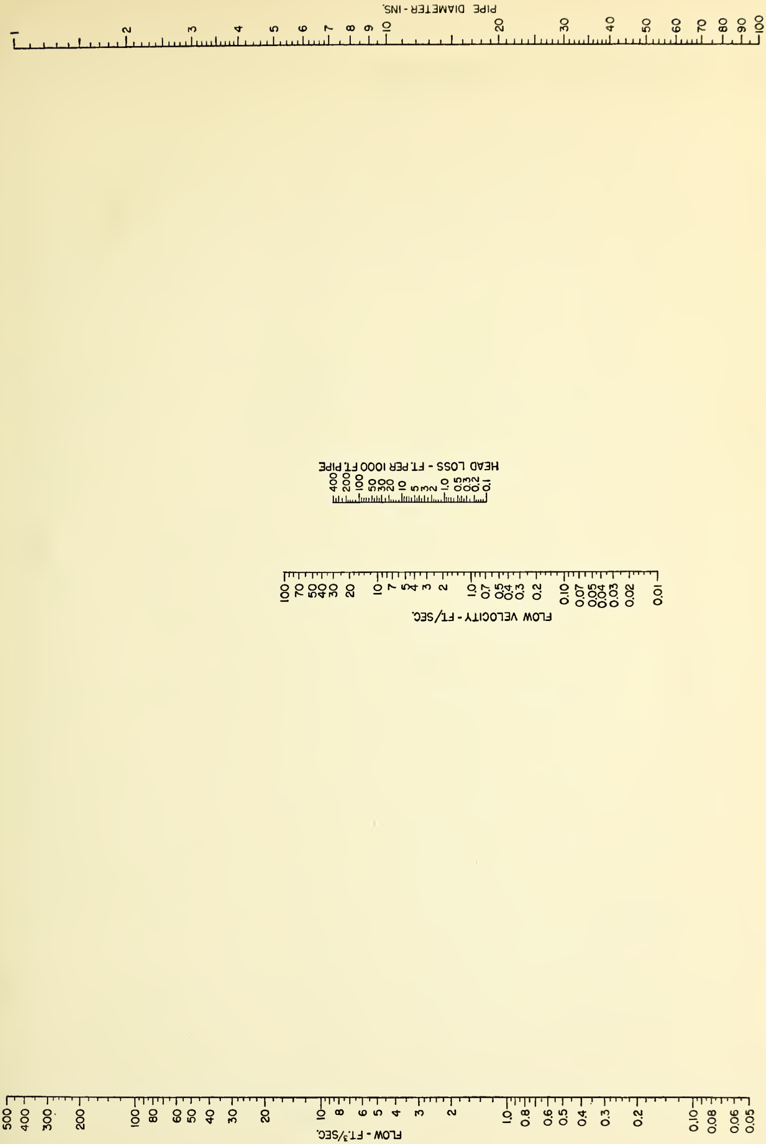
By way of illustrating the basic use of the chart, the flow may be determined for a channel of metal, smooth lumber, or masonry, having a roughness coefficient of 0.012, where the hydraulic radius is 1.00 foot and the slope is 1 foot in 2,000, or 0.0005, the flow cross section being assumed as 9.00 square feet. A line established by the scale point for 0.012 on the *Roughness Coefficient* scale and 1.00 on the *Hydraulic Radius* scale gives an intercept on the blank axis which fixes one point on the second line. With this, the line is completely defined by the proper scale point, 0.0005, on the *Slope* scale, and this line intersects the *Flow Velocity* scale at a scale point of 2.80 feet per second (computed value 2.77). The third line, established by this point and the scale point for 9.00 square feet *Channel Area,* intersects the *Flow* scale at a scaling of 25.0 cubic feet per second (computed value 24.93).


CHANNEL AREA-FT.²
HYDRAULIC RADIUS-FT.
FLOW-FT.³/SEC.
FLOW VELOCITY-FT./SEC.
ROUGHNESS COEFFICIENT-(N)
SLOPE

# 34. Pipe Flow and Loss

**Method for the determination** of flow velocity and head loss in flow of water in varying quantities and in different sizes of pipe. Its use is recommended *only* for preliminary or approximate determinations, because, in trying to make the chart cover a large field, accuracy is sacrificed. For accurate determinations, it is recommended that solution be broken down into two stages by use of Charts 35 and 36. In the text relating to those charts will be discussed their mathematical basis, and the same basis applies as well to this chart.

**Procedure.** In using the chart, a straight line is established by locating the scale points on the appropriate scales for any two of the variables involved: flow quantity, pipe diameter, flow velocity, and head loss. A straight line through the two known scale points will then intersect the other scales at scale points indicating the true values for those other two variables.


PIPE DIAMETER - INS.
1 2 3 4 5 6 7 8 9 10 20 30 40 50 60 70 80 90 100
HEAD LOSS - FT. PER 1000 FT. PIPE
400 200 100 50 30 20 10 5 3 2 1.0 0.5 0.3 0.2 0.1
FLOW VELOCITY - FT./SEC.
100 70 50 40 30 20 10 7 5 4 3 2 1.0 0.7 0.5 0.4 0.3 0.2 0.10 0.07 0.05 0.04 0.03 0.02 0.01
FLOW - FT.³/SEC.
500 400 300 200 100 80 60 50 40 30 20 10 8 6 5 4 3 2 1.0 0.8 0.6 0.5 0.4 0.3 0.2 0.10 0.08 0.06 0.05

# 35. Pipe Flow Velocity

**Method for the solution** of problems involving the relationship between pipe diameter, flow quantity, and flow velocity. Directly, the chart provides a determination of velocity when quantity and diameter are known, or it serves to determine pipe diameter where quantity and limiting velocity are known. Further, it may be used as the first step, in determining velocity, for other pipe-line problems, such as head loss and water-hammer surge-pressure rise, the subsequent steps for which are covered by later charts in this volume.

The formula upon which this chart is based is elementary, in that the flow quantity is

$$Q = VA$$

where $V$ is the velocity in feet per second, $A$ is the cross-sectional area of the pipe in square feet, and the quantity is in cubic feet per second. In another form, the equation is $V = Q/A$, and introducing the proper constants, it becomes $183.34Q/D^2$, where $D$ is the pipe diameter in inches.

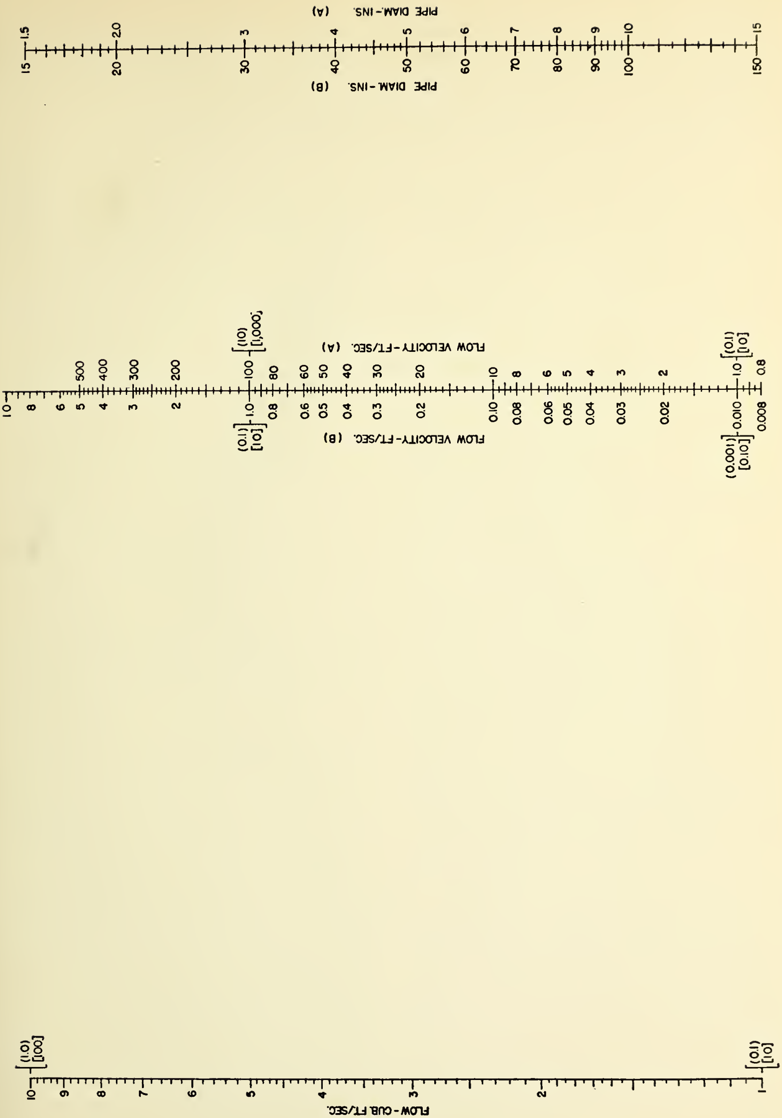
In the text accompanying Chart 34 (*Pipe Flow and Loss*), it was recommended that where any degree of accuracy was required in pipe-flow computations, the problem be divided into two stages. An examination of this chart reveals immediately the reason for this. This chart covers a wide range of values for the variables but with scales enormously expanded, making for far greater accuracy. To have achieved the same end on the earlier chart would have involved such complexities of scaling as to make its use more difficult and much more subject to errors in the use of auxiliary scalings.

It is repeated, therefore, that use of the earlier chart should be restricted to preliminary determinations or to those where only rough estimates are involved.

**Procedure.** To use the chart in determining velocity where quantity flow and pipe diameter are known, a straight line is established through the scale points corresponding to the known values on the scales for *Flow* and *Pipe Diameter*. At the intersection of this line with the *Flow Velocity* scale, the scale value is read for that figure. The *Pipe Diameter* and *Flow Velocity* scales have double scalings, and when the known diameter falls on the $A$ scaling for diameter, the $A$ scale for velocity is read: likewise, $B$ is used with $B$.

Where the *Flow* quantity falls outside the scale limits, recourse is had to the auxiliary scalings appearing at the ends of the scales. The actual quantity is multiplied by 10.0 or 0.1, as necessary, to obtain a figure that does fall within the scale limits. Then, the *Flow Velocity* scale reading is adjusted according to the auxiliary scaling figures, always keeping in mind the necessary consistency as to $A$ and $B$ scales.

For instance, determining the flow velocity with 2.50 cubic feet per second in a pipe having an internal diameter of 7.00 inches, a straight line through those respective scale points intersects the *Flow Velocity* axis at a value of 9.35 feet per second (true value 9.354), using the $A$


PIPE DIAM.-INS. (A)
PIPE DIAM.-INS. (B)
FLOW VELOCITY-FT/SEC. (A)
FLOW VELOCITY-FT/SEC. (B)
FLOW-CUB.FT/SEC.

scalings, of course. If the diameter is 70.0 inches, the $B$ scalings would be used, and the velocity would be 0.0935.

Further, had the quantity been 0.25 cubic foot per second, the velocity would have been 0.935 for $A$ or 0.00935 for $B$, and if 25.0, the results would be 93.5 and 0.935, respectively. Certain of these figures approach absurdity, but at least the principle is illustrated.

# 36. Pipe Flow Loss

**Method for the determination** of the head loss in friction in the flow of water in pipe. It is true that tabulations are readily available which present the losses with various pipe diameters and flows (velocities). However, it is seldom that interpolation is not necessary with such tabulations, and by reason of the exponential relationships of the variables, this is a tedious process, avoided by use of the chart.

It is also true that there have been numerous charts offered for solving this problem, but it is felt that this one, in combination with the one preceding, will offer certain advantages over all of them, even the one presented in this volume as Chart 34. In preceding discussions, it has been brought out that broad coverage of variation of variables in these charts has been achieved without the loss of accuracy consistently found in other charts that cover equal ranges.

There is one other departure, in this chart, from others that have appeared, in that this chart is based upon a formula differing somewhat from others used. Research over the past 20 years by this author has included observations on a number of long pipe lines, 8 to 100 miles in length, and in diameters from 8 inches to 6 feet. This has been in addition to shorter lines which included sizes down to 2 inches.

All of this study has shown conclusively that the head-loss variation with diameter is inversely with the 1.16 power, rather than the exponent as used in many formulas. The variation of exponent in the various formulas ranges from the 1.1 to the 1.33 power.

As to variation of loss with velocity, most formulas use a basis of a variation with the square of the velocity, with the exception of the 1.9 power used in one formula. The researches referred to here indicate that the 1.9 power is correct over a wide range of conditions. The variations include differences in pipe material, with tests on wood, concrete, cast-iron, concrete-lined, and steel pipe lines.

The formula upon which this chart is based gives the head loss, in feet, per 1,000 feet of pipe length as

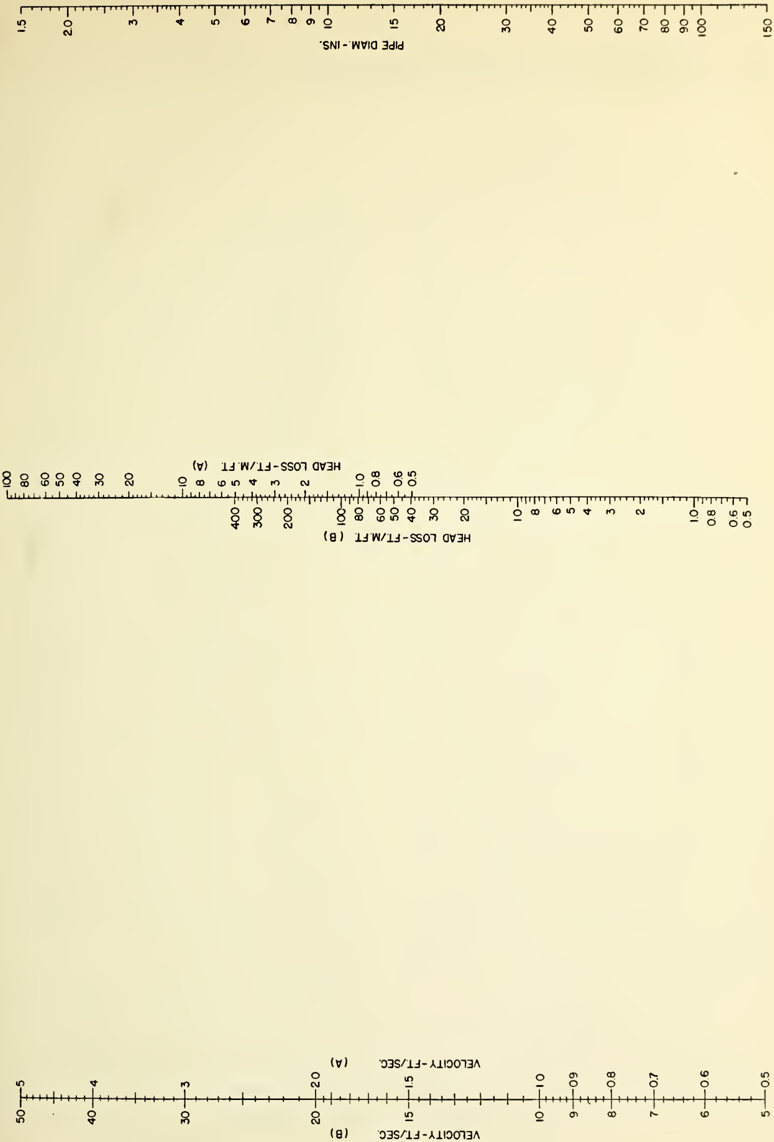
$$H = 6.2V^{1.9}/D^{1.16}$$

where $V$ is flow velocity in feet per second and $D$ is the internal diameter of the pipe in *inches.* It has been found to apply to lines in reasonably smooth internal condition. Manufacturers of the various types of pipe give various factors for determination of the change in loss with age. In the case of concrete, concrete-lined, or wood pipe, the loss may decrease slightly with the slimy film that is formed, whereas rusting and tuberculation will increase the loss in other types.

**Procedure.** To use the chart, it is necessary only to establish a straight line by the scale points on *Pipe Diameter* and *Velocity* scales. The intersection of this line with the *Head Loss* scale then gives the loss in feet per 1,000 feet. It should be noted that the *Velocity* axis is double-scaled, with scales noted as $A$ and $B$, in order to double the (logarithmic) scale of this variable.

For this reason, the *Head Loss* axis is likewise double-scaled, *A* and *B*, and in each case, the scalings of like designation must be used, *A* with *A* and *B* with *B*.

By way of illustration, the loss in a pipe of 2-inch true internal diameter with flow at 1.50 feet per second is determined by a line through 2.00 on the *Pipe Diameter* scale and 1.50 on the *A Velocity* scale. The intersection of this line with the *Head Loss* scale *A* gives the loss at 6.5 feet per 1,000 feet of pipe, which is reasonably in agreement with other charts and with figures presented by available tables.


PIPE DIAM.-INS.
HEAD LOSS-FT/M.FT (A)
HEAD LOSS-FT/M.FT (B)
VELOCITY-FT/SEC. (A)
VELOCITY-FT/SEC. (B)

# 37. Pressure Wave Velocity

**Method for the determination** of this characteristic of pipe lines. It is the necessary first stage in any study of surges in pipe lines, since from the result obtained from this chart can be found the maximum pressure rise resulting from instantaneous shutoff or its equivalent, for varying original velocities. The next stage is solved by means of the chart immediately following, and the next one, then, takes up cases of shutoff other than instantaneous.

In this chart and the two immediately following, all having to do with the solution of water-hammer surge problems, solution is based upon the elastic-water-column theory, assuming elasticity in the water itself as well as in the enclosing conduit. Other theories are frequently followed in such problems, and there are a number of approximate methods for solution, usually used in an effort to reduce the time otherwise required in solution. However, these other theories and methods have grave shortcomings and may lead to dangerously erroneous conclusions. This subject was very ably treated by Ray S. Quick in a paper prepared for the A.S.M.E. spring meeting of 1927, so further discussion is not necessary in this volume.

The basic formula upon which the chart is based is that wherein the wave velocity, in feet per second, is

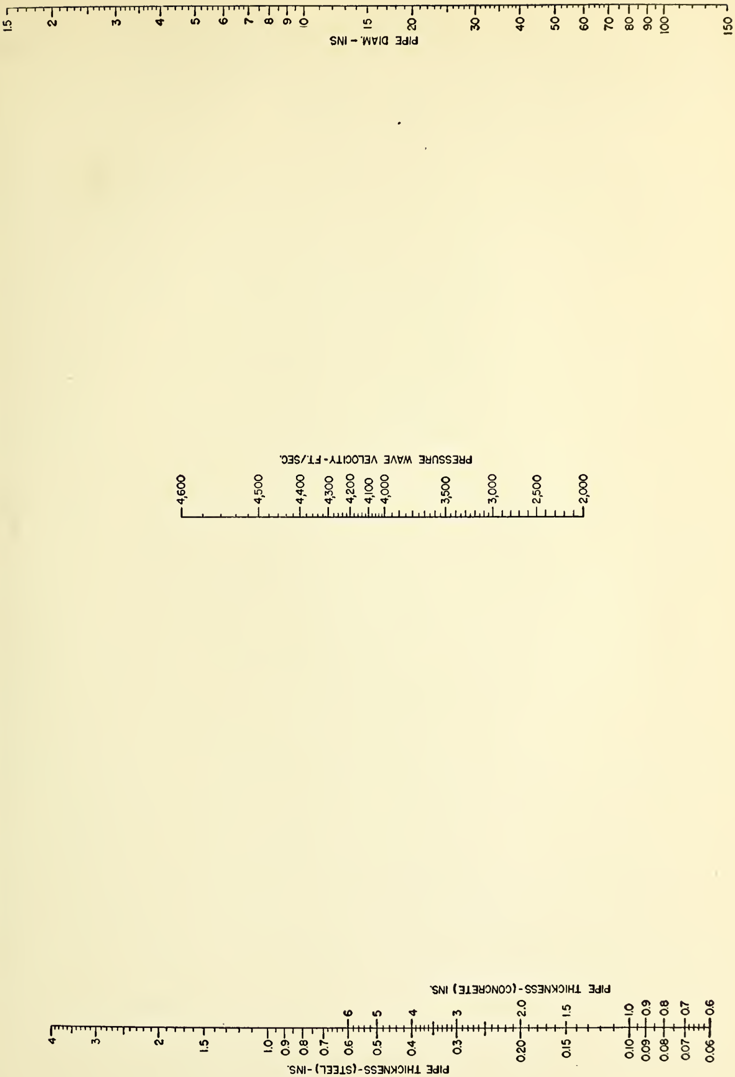
$$a = \frac{4{,}660}{\sqrt{1 + (KD/Et)}}$$

where $K$ is the bulk modulus of water = 294,000 pounds per square inch, $D$ is the pipe diameter in inches, $E$ is the modulus of elasticity of the pipe material in pounds per square inch, and $t$ is the pipe wall thickness in inches. $K$ is a constant, of course, and the terms $E$ and $t$ are combined in a single axis on the chart for purposes of simplification. Thus, there are two primary scalings, one where the material is concrete and the other where it is steel. An auxiliary scaling is also indicated for cast iron, each scale value on the steel scale representing twice that thickness for cast iron.

Studies by this author indicate that pipes of composite materials, such as concrete-lined steel or cast iron, or steel-banded wood-stave pipe, will have wave velocities more nearly those of the inner material than those of a computed combined modulus. There is little experimental data in this field, and in any case, manufacturers' guarantees must be used as the basis for any important studies involving such pipe, rather than relying upon this chart.

**Procedure.** To use the chart, it is necessary only to establish a straight line by the scale points on *Pipe Thickness* and *Pipe Diameter* scales. The intersection of this line with the *Pressure Wave Velocity* scale will be at the scale point for the velocity in feet per second.

By way of illustration, the wave velocity in a steel pipe line of 15.0 inches diameter and 0.60-inch thickness is determined by a line through 15.0 on the *Pipe Diameter* scale and 0.60 on the *Pipe Thickness* (*Steel*) scale. The intersection of this line


PIPE DIAM. - INS.
PRESSURE WAVE VELOCITY - FT./SEC.
PIPE THICKNESS - (CONCRETE) INS.
PIPE THICKNESS - (STEEL) - INS.

with the *Pressure Wave Velocity* scale is at the scale point corresponding to 4,160 feet per second (computed value 4,168).

It might be noted that in so far as wave velocity is concerned, the condition would be the same for a 15-inch cast-iron pipe 1.20 inches thick, or a 15-inch concrete pipe 6.0 inches thick. The latter approaches absurdity but illustrates the point.

# 38. Surge-pressure Rise

**Method for the determination** of surge pressures where flow shutoff is not instantaneous or equivalent thereto. The word "equivalent" applies, of course, to shutoff, assumed as taking place uniformly, over a period of time greater than the natural period of the pipe line (the time required for a surge-pressure wave to travel back to the end of the line and be reflected back to the point of shutoff).

The determination of pressure rise in less than the natural period of the line is a fairly simple matter, but for slower closure, the solution is a long, involved, and arduous task. The maximum pressure may occur after several cycles, and since it is the result of summations of reflections of waves from the partial shutoff in previous cycles, there is no short cut, the problem must be carried out step by step.

It is true that Allievi (*Theory of Water Hammer,* translated by E. E. Halmos, 1925), in his studies of water hammer, designed a chart for solving such problems, but interpolation, usually necessary, is difficult, and close results are not possible. Further, in a paper prepared for presentation at the 1927 meeting of the A.S.M.E., Ray S. Quick presented a chart of different design which, using a slightly different base form, made greater accuracy possible.

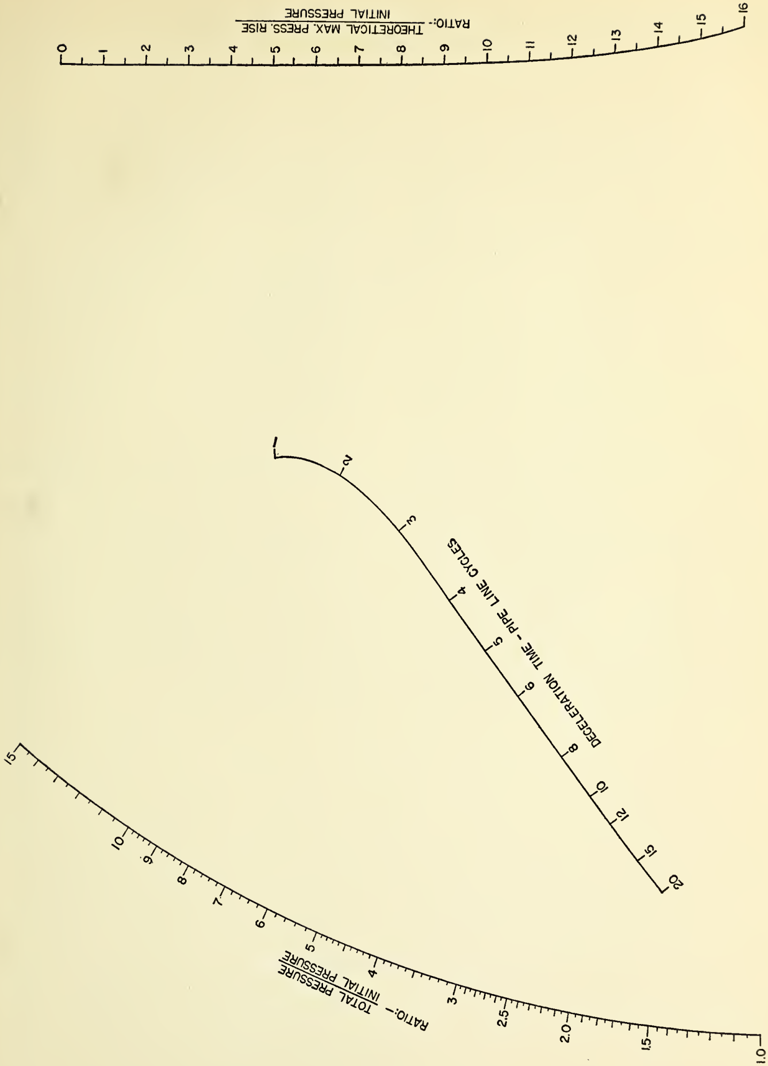
However, the chart presented here, when used in combination with the ones preceding and following, appears to be simpler in operation and more accurate in results than either of those mentioned above. Chart 37 having given the value of the pressure wave velocity in a line, Chart 39 then gives the figure for the maximum pressure rise possible for closure from any initial velocity. This chart now makes possible determination of the pressure increase with shutoff at a uniform but slower rate.

**Procedure.** Before proceeding to the use of this chart, it is necessary to make two very simple arithmetical computations based upon the results obtained from Charts 37 and 39. First, from the pressure wave velocity and the known length of pipe line, the natural period of the pipe line is determined, as mentioned in connection with Chart 39. Then the given length of time for uniform closure is reduced from actual chronological time units to number of pipe-line cycles.

Next, the ratio of maximum pressure rise (as determined by Chart 39) to the initial pressure is computed as a decimal figure. Then the problem is brought to this chart. A straight line is established by the scale points corresponding to the figures computed as described above, on the *Deceleration Time* and *Ratio Maximum Pressure/Initial Pressure* scales. The intersection of the prolongation of this line with the third scale, at the left of the chart, will be at a scale value representing the *Ratio Total Pressure/Initial Pressure,* which is the ratio of the maximum pressure, including superimposed surge, to the initial pressure.

By way of illustration, but omitting preliminary computations, the maximum pres-

sure may be determined for a situation where initial pressure is equal to 100 feet and the line and initial velocity are such that instantaneous shutoff would give a pressure *rise* of 400 feet, the closure time now to be in a period of 10 cycles. The ratio of maximum pressure *rise* to initial pressure is 4.00, and a line is fixed through this scale point and the scale point for 10.0 cycles, and this line, prolonged, intersects the third scale at a scale value of 1.224. The maximum pressure, including *initial pressure* and *superimposed* surge, is then $1.224 \times 100$ (initial pressure), or 122.4 feet.



RATIO:- THEORETICAL MAX. PRESS. RISE / INITIAL PRESSURE
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
DECELERATION TIME - PIPE LINE CYCLES
1
2
3
4
5
6
8
10
12
15
20
RATIO:- TOTAL PRESSURE / INITIAL PRESSURE
1.0
1.5
2.0
2.5
3
4
5
6
7
8
9
10
15

# 39. Maximum Surge-pressure Rise

**Method for the determination** of the magnitude of the pressure head superimposed upon normal pressure by instantaneous shutoff of flow. The equation upon which the chart is based is that the head rise, in feet, is

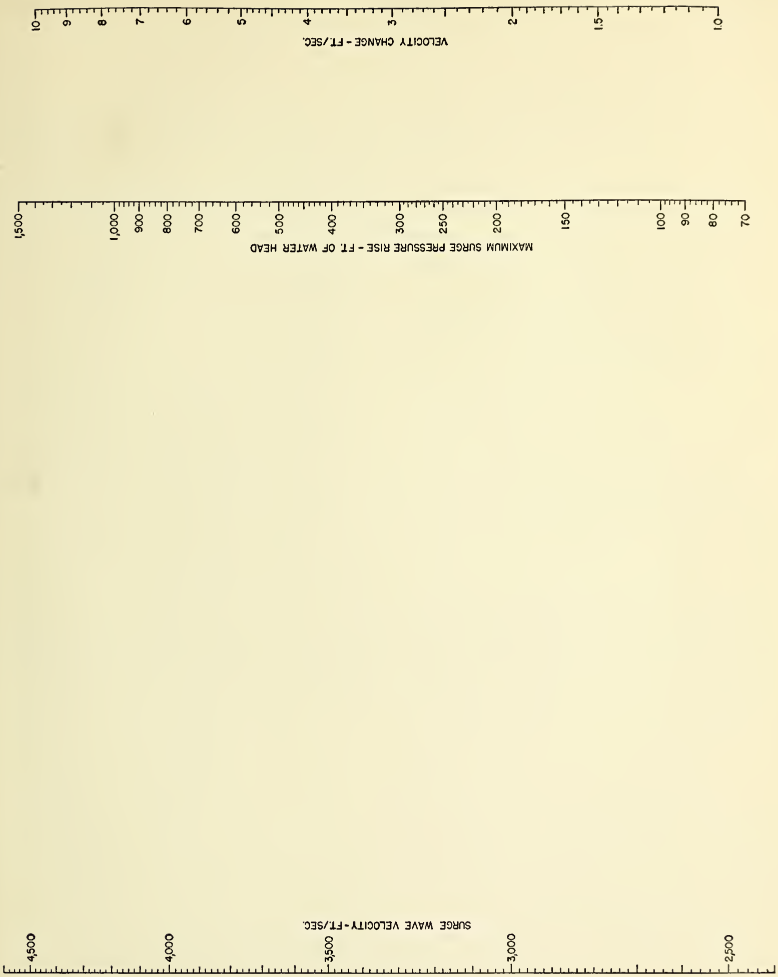
$$h_0 = aV_0/g$$

where $a$ is the pressure wave velocity in feet per second (determined by Chart 37), $V_0$ is the initial velocity of water flow in the line in feet per second, and $g$ is gravitational acceleration, here taken as 32.162 feet per second per second.

This equation and chart serve for solution not only for instantaneous shutoff of flow but for any equivalent condition. In any line, the natural period of the line will determine the conditions equivalent to instantaneous shutoff. Shutoff in any length of time less than that required for the pressure wave to travel to the inlet and back to the point of shutoff will be equivalent to instantaneous closure. With pipe-line length = $L$ known (in feet) and the wave velocity $a$ determined from Chart 37 in feet per second, the natural period of the line will be $2L/a$ seconds.

**Procedure.** To use the chart, it is necessary only to establish a straight line by the scale points on *Velocity Change* and *Surge Wave Velocity* scales. The intersection of this line with the *Maximum Surge Pressure Rise* scale will be at the scale point for such rise, measured in feet of water.

By way of illustration, the pressure rise with instantaneous shutoff of a flow at 8.00 feet per second in a line having a pressure wave velocity of 3,500 feet per second is determined by a line through 8.00 on the *Velocity Change* scale and 3,500 on the *Surge Wave Velocity* scale. The intersection of this line with the *Maximum Surge Pressure Rise* scale is at the scale point corresponding to 870 feet of water head (computed value 870.6).


VELOCITY CHANGE - FT./SEC.
MAXIMUM SURGE PRESSURE RISE - FT. OF WATER HEAD
SURGE WAVE VELOCITY-FT./SEC.

# 40. Pipe or Tank Volume

**Method for the determination** of the volume or content of cylindrical enclosures. The scalings are based upon dimensions in inches, but other units may be used if carried through consistently. However, the reference scale, *Gallons per Foot of Length,* can be used only when the units are inches. This scaling is handy for reference, although it is not carried through the chart.

The mathematical basis for the chart is obvious, of course, in that

$$\text{Area} = A = \pi D^2/4$$

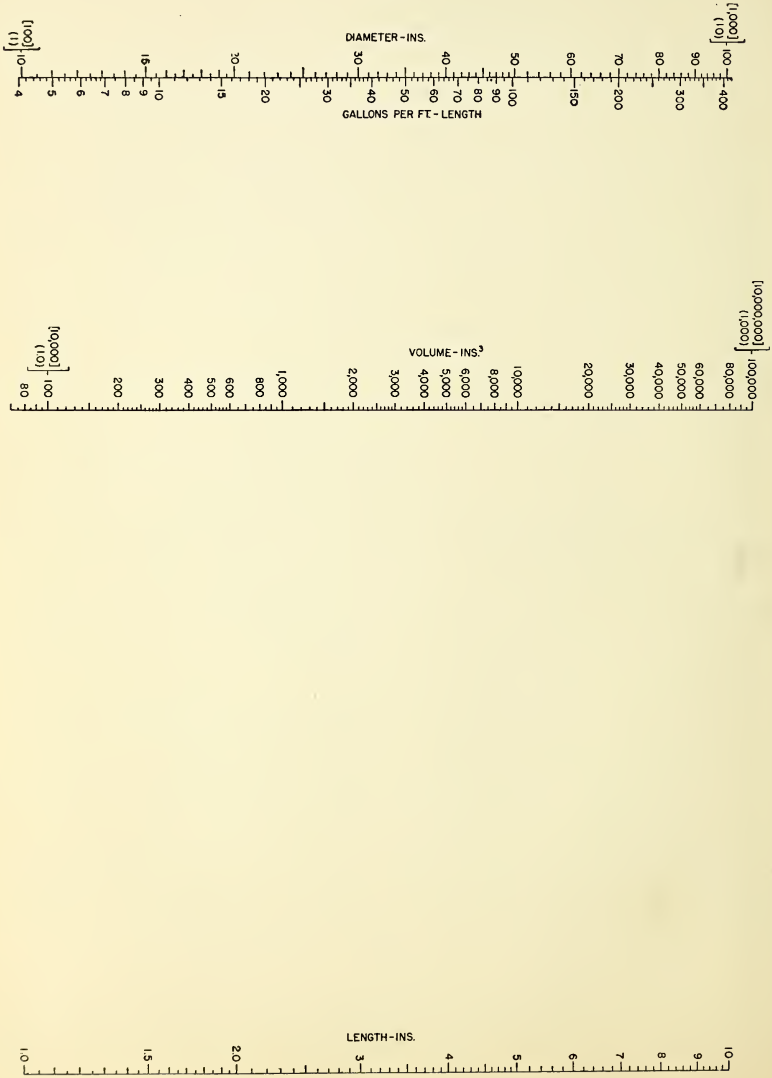
and

$$\text{Volume} = V = LA$$

where $D$ is diameter and $L$ is length. Gallonage per foot of length for varying diameter is based upon 231 cubic inches per gallon.

**Procedure.** To use the chart, it is necessary only to establish a straight line by the scale points for the known values on the *Diameter* and *Length* scales. The scale point at which this line intersects the *Volume* scale then gives that value. Where the given dimensions fall beyond the primary scaling limits, they should be multiplied by such powers of 10 as will bring them within those limits. Auxiliary scalings appear at the ends of the *Diameter* and *Volume* scales to indicate the relative variation between these variables. The variation with *Length* is, obviously, a direct one.

By way of illustrating use of the chart, the volume is determined for a cylinder of 12.0-inch diameter and 30.0 inches length, by establishing a straight line through 12.0 on the *Diameter* scale and *3.00* on the *Length* scale. This intersects the *Volume* scale at a scale point corresponding to 340 cubic inches. Since the length scaling used was one-tenth of the true length, the actual volume will be ten times the figure read, or 3,400 (true value 3,393). If of interest, the gallonage per foot may be read opposite 12.0 on the *Diameter* scale as 5.88 (true value 5.875).

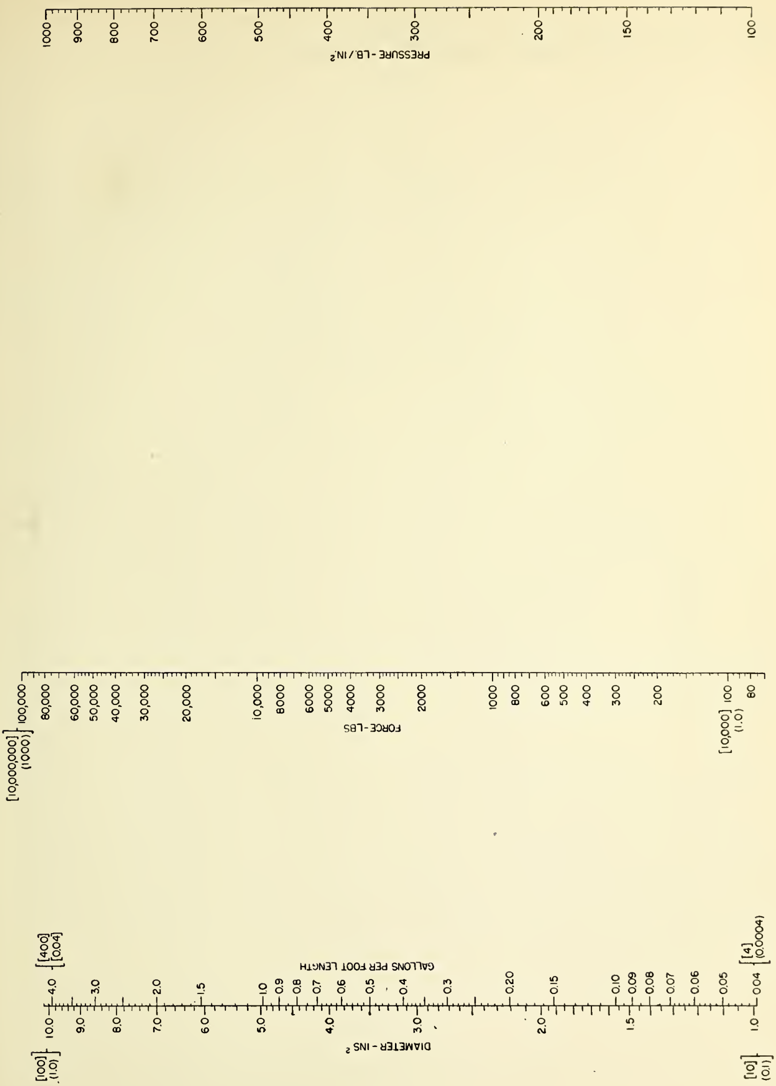

DIAMETER - INS.
GALLONS PER FT - LENGTH
VOLUME - INS.$^3$
LENGTH - INS.

# 41. Hydraulic-cylinder Force

**Method for the determination** of the force resulting from pressure in a cylinder against a piston. The total force is, of course, equal to the pressure times the area, and, the chart being scaled upon inch units, pressures are in pounds per square inch, diameters are in inches, and areas in square inches. For convenience, the *Diameter* scale also carries a scaling to indicate *Gallons per Foot Length.*

**Procedure.** To use the chart, it is necessary only to establish a straight line by the scale points for the given values on the *Diameter* and *Pressure* scales. The scale point at which this line intersects the *Force* scale then gives that value. Where the given figures fall beyond the primary scaling limits, they should be multiplied by such powers of 10 as will bring them within those limits. Auxiliary scalings appear at the ends of the *Diameter* and *Force* scales to indicate the relative variations between these variables. The variation with *Pressure* is, obviously, a direct one.

By way of illustrating use of the chart, the force is determined for a ram of 2.50-inch diameter under 800 pounds per square inch pressure, by establishing a straight line through 2.50 on the *Diameter* scale and 800 on the *Pressure* scale. This intersects the *Force* scale at a point corresponding to 3,930 pounds (computed value 3,926.96). If of interest, the gallonage per foot of cylinder length may be read opposite 2.50 on the *Diameter* scale as 0.255.


PRESSURE - LB./IN.²
1000 900 800 700 600 500 400 300 200 150 100
FORCE - LBS
[10,000,000] (1000) 100,000 80,000 60,000 50,000 40,000 30,000 20,000 10,000 8000 6000 5000 4000 3000 2000 1000 800 600 500 400 300 200 100 80 [10,000] (1.0)
DIAMETER - INS ²
[100] (1.0) 10.0 9.0 8.0 7.0 6.0 5.0 4.0 3.0 2.0 1.5 1.0 [10] (0.1)
GALLONS PER FOOT LENGTH
[400] [0.04] 4.0 3.0 2.0 1.5 1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.20 0.15 0.10 0.09 0.08 0.07 0.06 0.05 0.04 [4] (0.0004)

# 42. Horizontal-tank Content

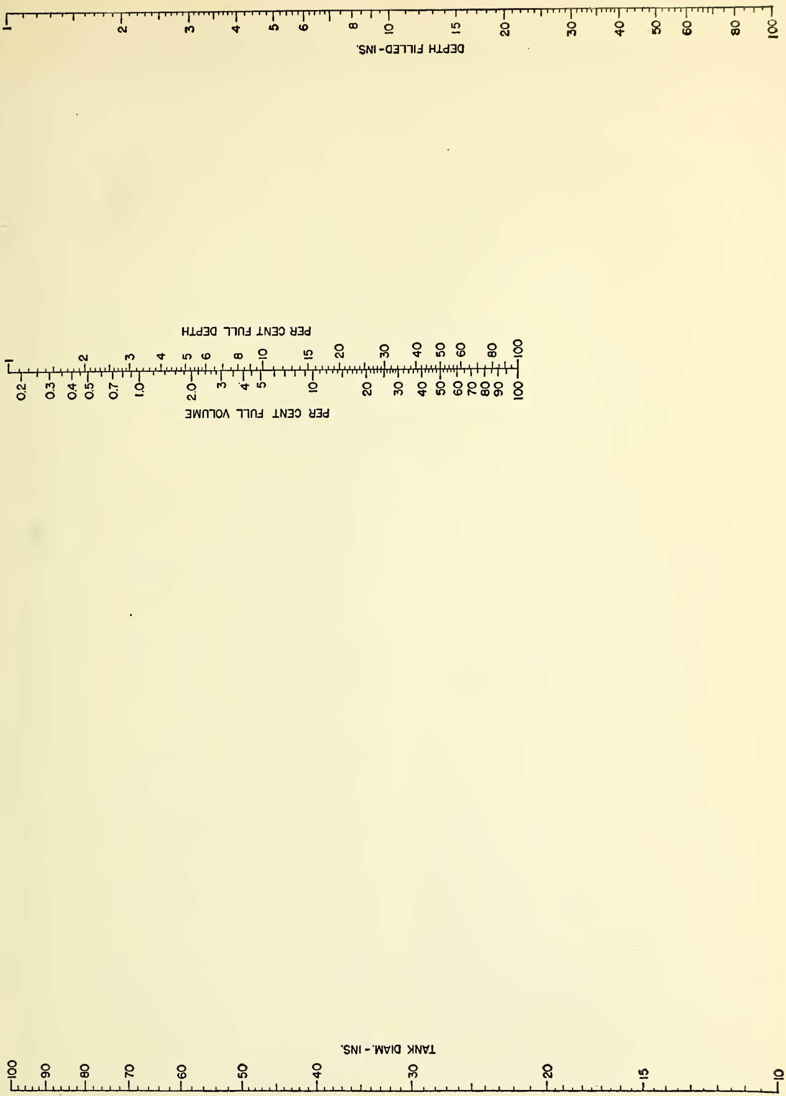
**Method for the determination** of the content of partially filled, horizontal, cylindrical tanks of varying diameters and with varying depths of fill. This particular chart gives the result as a percentage of the volume when filled, but, in combination with Chart 40, the over-all solution involves but one simple computation aside from the use of the charts.

Charts presenting the relationship between percentage depth and percentage fill are not commonly available, and even when they can be used, interpolation, necessary more often than not, is difficult by reason of the nonlinear relationship. With a graphical representation, however, interpolation is an easier task. But where tables or charts are not available, the next choice is the use of tables of chords, segment areas, etc., of circles, but here again interpolation is usually necessary in each of the various steps, and the final step is the multiplication of several factors.

**Procedure.** To use the chart, it is necessary only to establish a straight line by the scale points for the known values on the *Tank Diameter* and *Depth Filled* scales. The scale points at which this line intersects the third axis are then the values for *Per Cent of Full Depth* and *Per Cent of Full Volume*. Having used Chart 40 to determine the full volume, determination of the partial volume is then a mere multiplication. Where the given values fall outside the scale limits of the chart, the true values should each be multiplied by such a power of 10.0 as will bring figures within the scaling ranges. The result, however, will be the direct reading on the central axis.

To achieve a greater measure of accuracy under conditions where the depth filled is more than 50 per cent, it is suggested that the *unfilled* portion of total depth be used, which brings the final determination into a portion of the scale that is more expanded. The true percentage filled is then the difference between the observed figure and 100 per cent.

By way of illustrating the use of the chart, the per cent of full volume for a tank diameter of 15 inches filled to a depth of 13 inches is determined by first changing the figures to cover *unfilled* depth, *i.e.,* 2 inches. Then, a straight line through the scale points of 15.0 on the *Tank Diameter* scale and 2.00 on the *Depth Filled* scale intersects the third axis at points giving 13.3 as the *Per Cent of Full Depth* and 8.0 as the *Per Cent of Full Volume.* The actual figures then, again reversing, are 86.7 per cent of full depth and 92.0 per cent of full volume (true value 92.09).


DEPTH FILLED - INS.
PER CENT FULL DEPTH
PER CENT FULL VOLUME
TANK DIAM. - INS.

# 43. Cylindrical-tank and Pipe Strength

**Method for the determination** of the circumferential stress in the shell of a cylindrical tank or of a pipe under pressure, or knowing that factor, it provides for finding the maximum allowable internal pressure. As a matter of fact, as among the four variables relating to a cylindrical-tank shell or pipe wall, *i.e.,* internal pressure, limiting stress in the shell, internal diameter, and shell thickness, where any three are known or fixed, the chart serves to find the fourth.

The mathematical expression upon which the chart is based is, of course, the conventional one, wherein

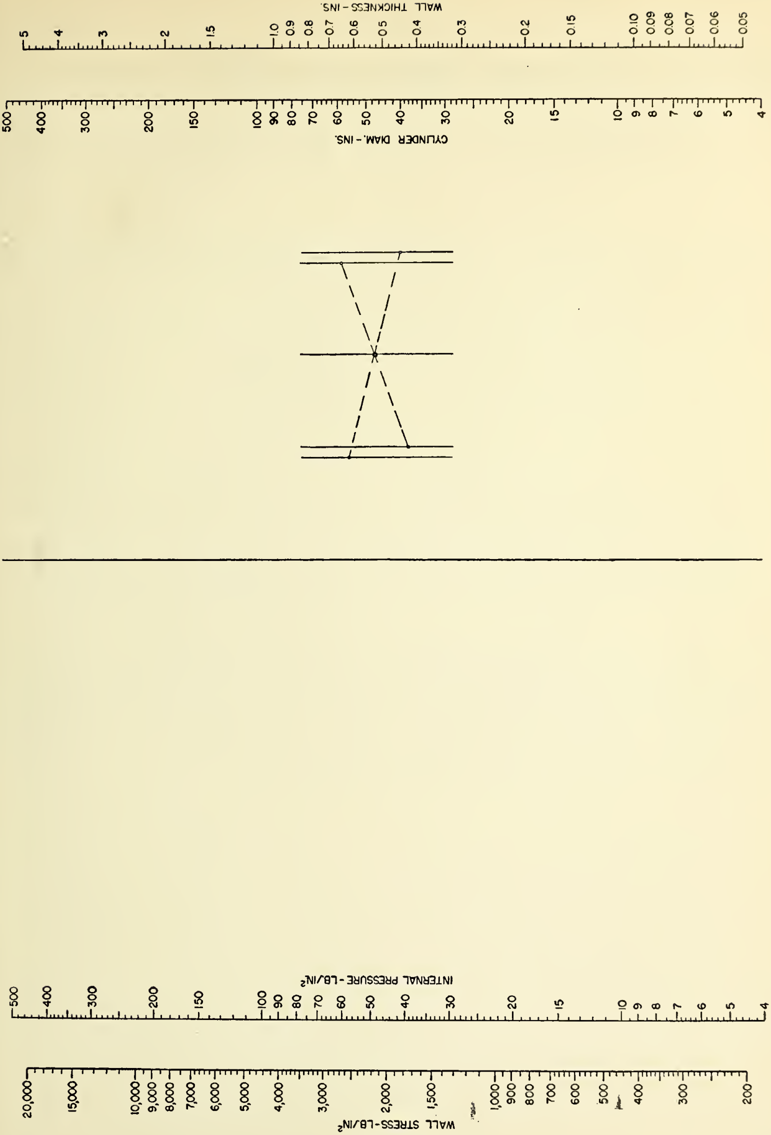
$$PD = 2ST$$

where $P$ is the internal pressure in pounds per square inch, $D$ is the internal diameter in inches, $S$ is the unit stress in the shell material in pounds per square inch, and $T$ is the shell or wall thickness in inches. A guide diagram on the chart indicates the manner in which the variables are related.

**Procedure.** By way of further clarification, it may be explained that straight lines through (1) proper scale points for *Internal Pressure* and *Cylinder Diameter* and (2) proper scale points for *Wall Stress* and *Wall Thickness* will both intersect the blank axis at the same point, if the values meet the requirements of the equation. Thus, to use the chart, knowing any three of the four variables, the intersection of the one straight line, either (1) or (2), that is fixed by known quantities is laid out to determine the intersection with the central, blank axis. This point of intersection, together with the scale point corresponding to the other known quantity, determines a second straight line, whose intersection with the last scale gives the scale value for the fourth quantity that will satisfy the equation.

By way of illustrating the use of the chart, assume a cylinder of 8.00-inch internal diameter, with an internal pressure of 300 pounds per square inch, the limiting value of stress in the shell being 2,000 pounds per square inch. First, a line is established through the scale points of 300 on the *Internal Pressure* scale and 8.00 on the *Cylinder Diameter* scale, and the intersection of this line with the central, blank axis is noted. A second line is then established through this point and the scale point for 2,000 on the *Wall Stress* scale, and the intersection of this line with the *Wall Thickness* scale is noted at a scale point of 0.600 inch (true computed value 0.596).

To illustrate further, assume a cylinder of 0.75-inch wall thickness, with limiting stress set as 10,000 pounds per square inch, the diameter being 50.0 inches, the problem being to determine the limiting internal pressure. In this case, it is the group (2), *Wall Stress* and *Wall Thickness,* being known, that determine the first straight line, set by the scale points for those two factors, and fix an intercept on the blank axis. A second line through this intercept point and the scale point for the third known value, *Cylinder Diameter,* is then established, and its intercept on the last scale, *Internal Pressure,* is read as 305, approximately (computed value 300).


WALL THICKNESS - INS.
CYLINDER DIAM. - INS.
INTERNAL PRESSURE - LB/IN²
WALL STRESS-LB/IN²

# 44. Jet Diameter and Theoretical Water Horsepower

**Method for the determination** of any two of the variables of water quantity, net head (pressure) at the jet orifice, jet diameter, and theoretical water horsepower, where the other two are known. Without going into the derivation of the formulas involved, it may be stated that the two basic equations are

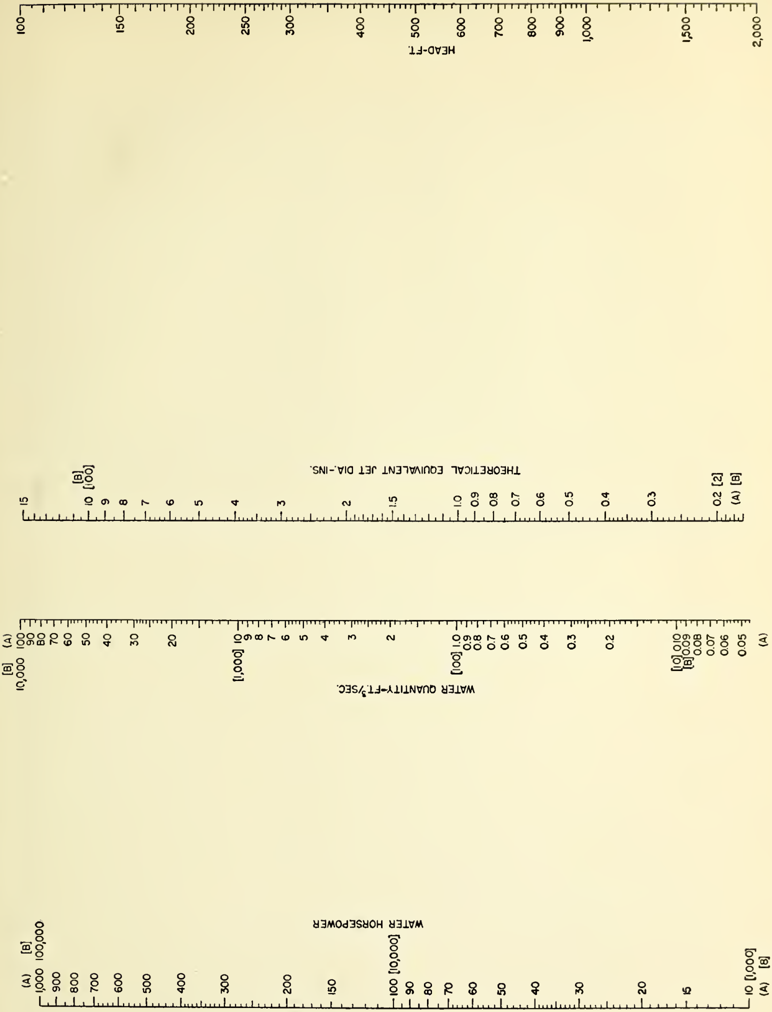
$$WHP = \frac{QH}{8.81} \quad \text{and} \quad D = \frac{14.188 WHP^{0.5}}{H^{0.75}}$$

where $WHP$ is the theoretical water horsepower of the jet, $Q$ is the water quantity in cubic feet per second, $H$ is the net effective head in feet, and $D$ is the jet diameter in inches.

**Procedure.** In using the chart, it is necessary only to establish a straight line through the scale points corresponding to the known values of any two of the variables, upon which the scale values for the other two may be read directly. Any two may be known and thereby establish the line which determines the values of the other two.

By way of illustration, assume a 3.00-inch jet diameter under a net effective head of 100 feet. A line through the scale points corresponding to these two values intersects the other scales at scale points indicating a discharge of 3.68 cubic feet per second (true value 3.66) and an output of 42.0 theoretical water horsepower (true value 41.64).

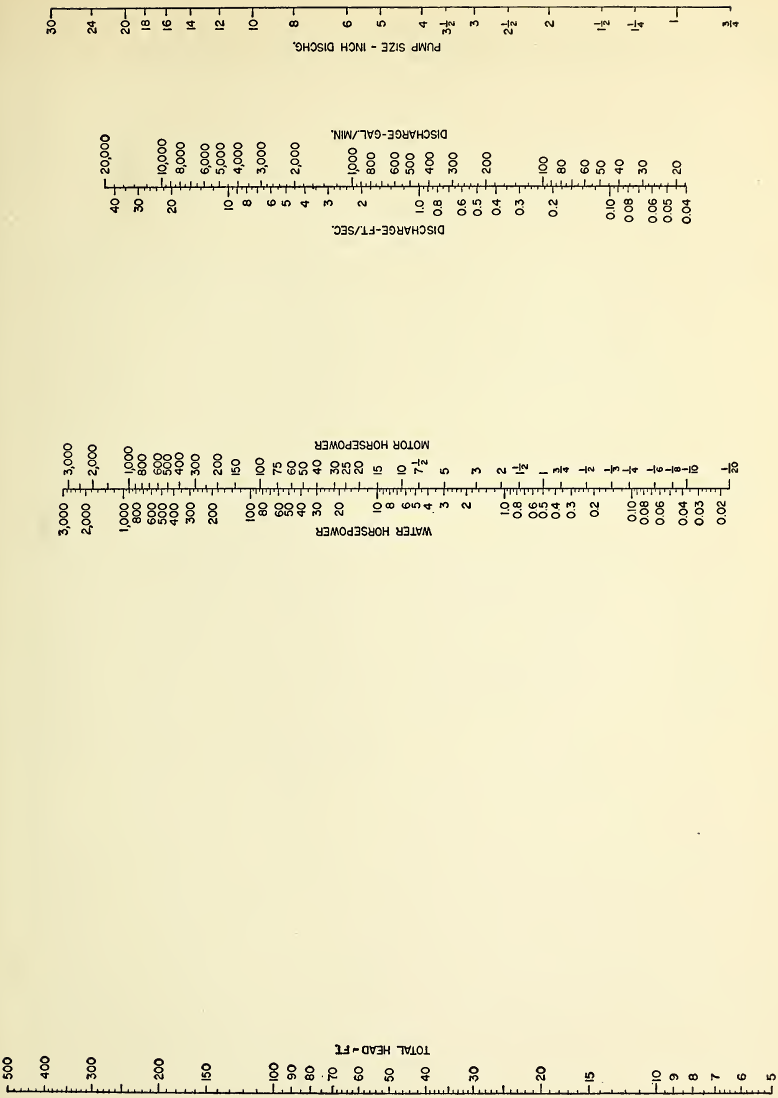
Using the chart in another way, assume a water quantity of 2.00 cubic feet per second under a head of 150 feet. The scale points corresponding to these values establish a line which intersects the other scales at scale points indicating 2.00 as the jet diameter (true value), and 34.1 as the theoretical horsepower developed (true value 34.0).


HEAD-FT.
THEORETICAL EQUIVALENT JET DIA.-INS.
WATER QUANTITY-FT³/SEC.
WATER HORSEPOWER

# 45. Pump Size and Horsepower

**Method for the determination** of preliminary values for pump size and power requirements. This chart is not, *by any means,* offered as an exact determination in either regard, since each manufacturer has his own design standards, wherein he balances many factors which have a bearing upon the size of connections and upon efficiency and, thereby, drive horsepower. However, the experience of this writer indicates that this chart will in almost every case approach a median of the offerings of many manufacturers, and it has been useful in laying out preliminary plans.

**Procedure.** No pretense is made for having any definite mathematical basis for the chart, but, as stated above, it provides a close approach to the median figures of many manufacturers. So without further discussion, it is stated that the use of the chart consists only in establishing a straight line through scale points corresponding to the fixed values for *Discharge* (cubic feet per second or gallons per minute) and *Total Head* in feet. The intercepts of this line on the *Pump Size* (discharge diameter in inches) and *Motor Horsepower* scales will then give reasonable approximations of these quantities for preliminary purposes. The scale value for *Water Horsepower* will be correct within the limitations of this type of computation.


PUMP SIZE - INCH DISCHG.
DISCHARGE-GAL./MIN.
DISCHARGE-FT./SEC.
MOTOR HORSEPOWER
WATER HORSEPOWER
TOTAL HEAD - FT

# 46. Hydraulic-turbine Specific Speed

**Method for the determination** of this basic characteristic for turbines of any of the general types and for a wide range in the variables. However, a departure from usual practice is made here. It has long appeared to this author that a true specific speed, or runner characteristic, must be based upon the water horsepower, rather than shaft output, because specific speed is a hydraulic rather than a mechanical characteristic.

The formula upon which the chart is based is the usual one, where specific speed in English units is

$$N_0 = NP^{0.5}/H^{1.25}$$

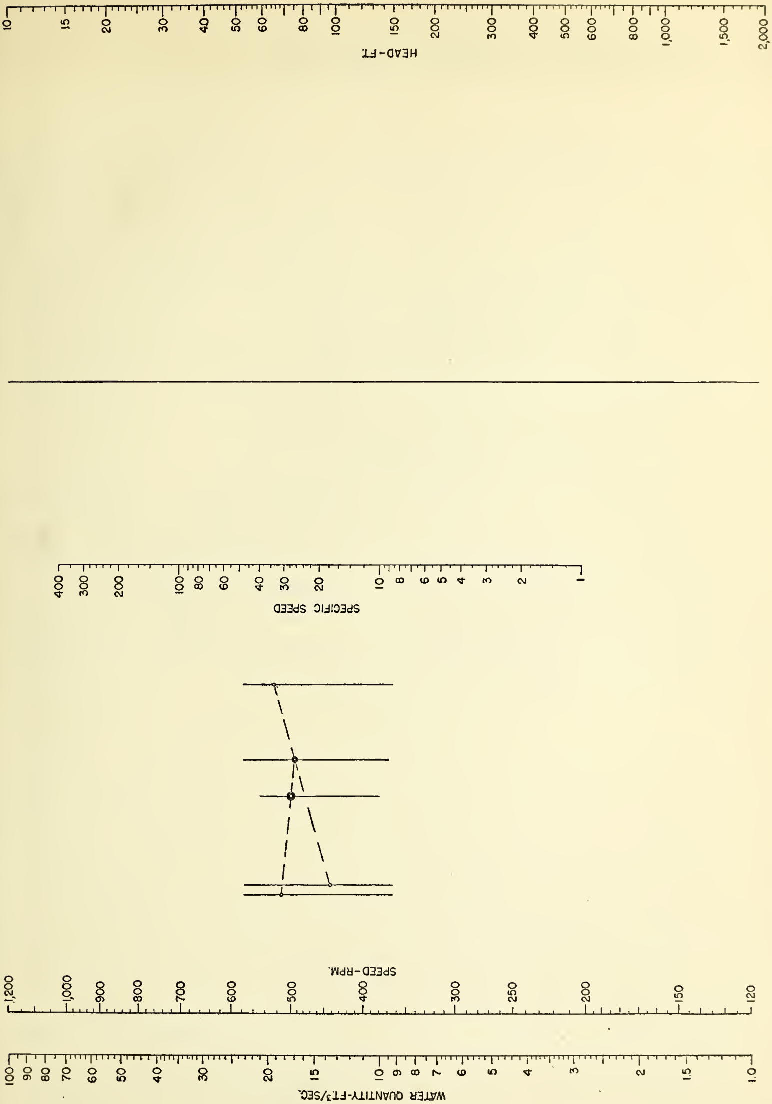
where $N$ is the rotational speed in revolutions per minute, $P$ is the horsepower, in this case being *water horsepower,* and $H$ is the net effective head in feet. But since $P = QH/8.81$, where $Q$ is the water quantity in cubic feet per second, the first equation is reduced to

$$N_0 = NQ^{0.5}/2.968H^{0.75}$$

and it is this form that is used as the basis for the chart.

**Procedure.** To use the chart, a straight line is established through the scale points for the known values on the *Speed* and *Head* scales, and the intercept of this line is noted on the blank axis. A second line is then established through this point and the correct scale point on the *Water Quantity* scale, and the intercept of this line with the *Specific Speed* scale will be at the scale point corresponding to the desired figure.

By way of illustration, the specific speed of a turbine operating under a 20.0-foot head at 1,000 revolutions per minute, with a flow of 16.0 cubic feet per second, may be determined. First, a line is fixed by the scale points of 20.0 on the *Head* scale and 1,000 on the *Speed* scale, and its intercept on the blank axis is noted. Then a line through this point and the scale point for 16.0 on the *Water Quantity* scale intersects the *Specific Speed* scale at a scale point of 142 (computed value 142.4).

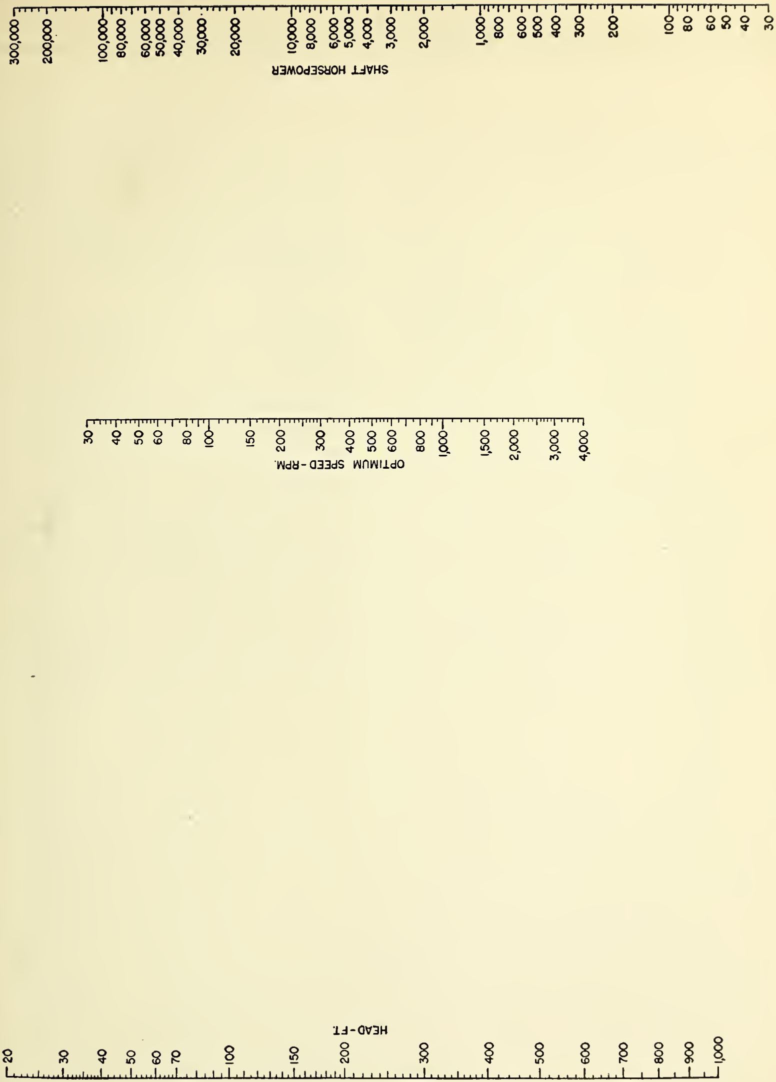

HEAD-FT.
SPECIFIC SPEED
SPEED-RPM.
WATER QUANTITY-FT³/SEC.

# 47. Hydraulic-turbine Speed

**Method for the determination** of preliminary values for the speed of the reaction (Francis) type of hydraulic turbine. This chart is not, by any means, offered as an exact determination, since each manufacturer has his own design standards and these vary. In addition, conditions of any particular installation may dictate departures from the optimum. However, the experience of this writer indicates that this chart, in almost every case, will approach a median of the specifications of various manufacturers. Thus, it has proved useful in the laying out of preliminary plans where exact designing is not required.

What has just been said regarding accuracy does not mean that the chart is unreliable, by any means. Several other charts for this same purpose have been seen by this author, and the results determined from them have been compared with this one for the actual design conditions for a total of 43 specially built turbines of different manufacturers. This chart gave a speed value within 15 per cent of the actual speed in 27 cases of the 43, whereas the three other charts so checked came within that limit in only 17, 20, and 18 cases, respectively. Further, this chart came within 5 per cent of the true value in 12 cases, whereas for the others the numbers were 9, 7, and 10, respectively.

**Procedure.** To use the chart, it is necessary only to establish a straight line through the scale points corresponding to the fixed values of *Head* and *Shaft Horsepower* on their respective scales. Then, at the scale point of the intersection of this line with the central *Speed* scale is read the desired value.

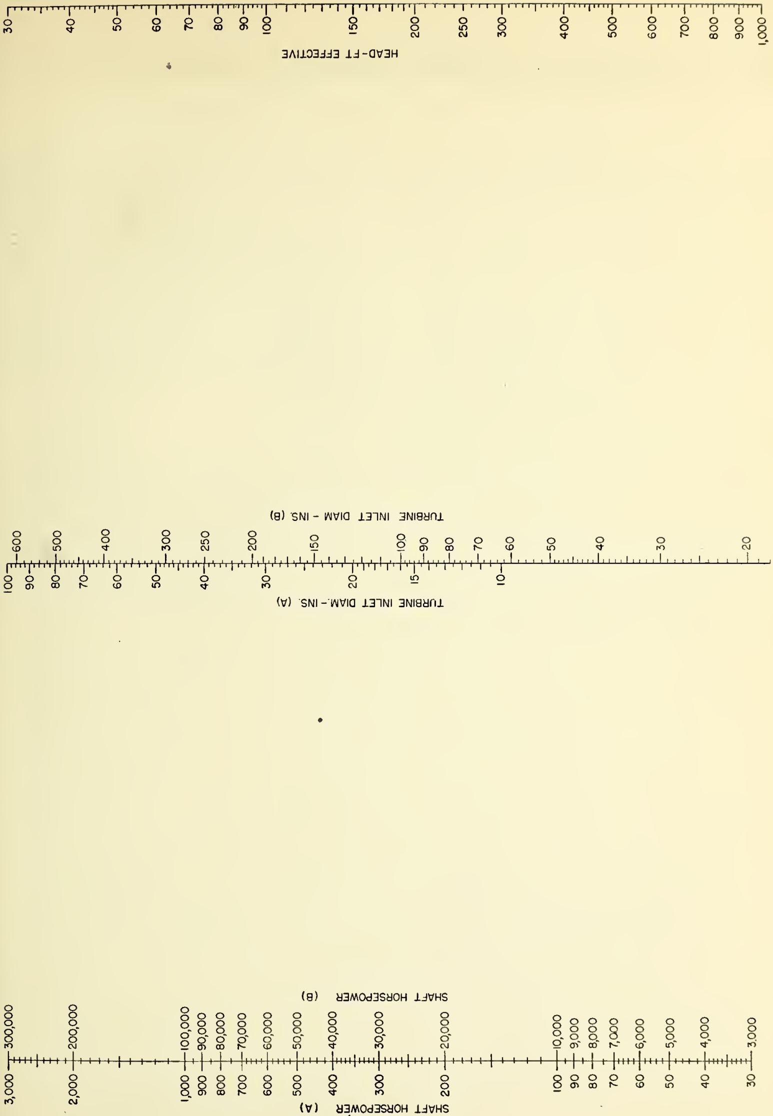

SHAFT HORSEPOWER
OPTIMUM SPEED-RPM.
HEAD-FT.

# 48. Hydraulic-turbine Inlet Diameter

**Method for the determination** of preliminary values for the inlet size of the reaction (Francis) type of hydraulic turbine. This chart is not, by any means, offered as an exact determination, since each manufacturer has his own design standards and these vary. In addition, conditions of any particular installation may dictate departures from the optimum. However, the experience of this writer indicates that this chart, in almost every case, will approach a median of the specifications of various manufacturers. Thus, it has proved useful in the laying out of preliminary plans where exact designing is not required.

What has just been said regarding accuracy does not mean that the chart is unreliable, by any means. Other charts for this same purpose have been seen by this author, and the results obtained from them have been compared with this one for the actual design conditions for a series of 30 specially built turbines. This chart gave figures that were within 15 per cent of the actual design figures in 23 of the cases, and within 5 per cent in 13 cases. The other charts achieved such approaches in less than two-thirds as many cases.

**Procedure.** To use the chart, it is necessary only to establish a straight line through the scale points corresponding to the fixed values of *Shaft Horsepower* and *Head* on their respective scales. Then, at the scale point of the intersection of this line with the central *Turbine Inlet Diameter* scale is read the desired value. It should be noted that the *Horsepower* axis is double-scaled, with scalings *A* and *B,* and whichever one is used for the horsepower value, the inlet size should be read on the inlet scale of like designation. Thus, for power values on the *A* scale, inlet size is read on the *A* scale, and similarly, *B* is used with *B*.


HEAD-FT EFFECTIVE
TURBINE INLET DIAM - INS. (B)
TURBINE INLET DIAM.- INS. (A)
SHAFT HORSEPOWER (B)
SHAFT HORSEPOWER (A)

# 49. Hydraulic-turbine Runner Diameter

**Method for the determination** of preliminary values for the runner size of the reaction (Francis) type of hydraulic turbine. This chart is not, by any means, offered as an exact determination, since each manufacturer has his own design standards and these vary. In addition, conditions of any particular installation may dictate departures from the optimum. However, the experience of this writer indicates that this chart, in almost every case, will approach the median of the specifications of various manufacturers. Thus, it has proved useful in the laying out of preliminary plans where exact designing is not required.

What has just been said regarding accuracy does not mean that the chart is unreliable, by any means. Other charts for this same purpose have been seen by this author, and the results determined from them have been compared with this one for the actual design conditions for a total of 43 specially built turbines. This chart gave figures for the runner diameter within 15 per cent of the actual design in all 43 cases and within 2 per cent in 23 cases. The best of the other charts came within 15 per cent in only 18 cases and within 5 per cent in only 8.

**Procedure.** To use the chart, it is necessary only to establish a straight line through the scale points corresponding to the fixed values of *Head* and *Speed* on their respective scales. Then, at the scale point of the intersection of this line with the central *Runner Diameter* scale is read the desired value.


SPEED-R.P.M.
RUNNER DIA-INS.
HEAD-FT.

# 50. Rainfall Intensity

**Method for estimating** the maximum expected rainfall intensity over various length intervals and for different localities. Of course, any really close predictions are possible only after study of records over many years in the particular locality under consideration. However, where such records are not available, this chart will provide a reasonable basis for the design of drainage works.

The equation upon which the chart is based is not one of the better-known ones; rather the chart is based upon studies of the formulas in use in various places. It was found that many cities use the expression that rainfall intensity, in inches per hour, is

$$I = C/T^{0.5}$$

where $C$ is an arbitrary constant found applicable to the particular location and $T$ is the length of the time interval, in minutes, over which the average intensity will continue. A correlation was found between the arbitrary constant and the average annual rainfall in the location, and using $R$ as the value of the mean annual rainfall in inches, the equation

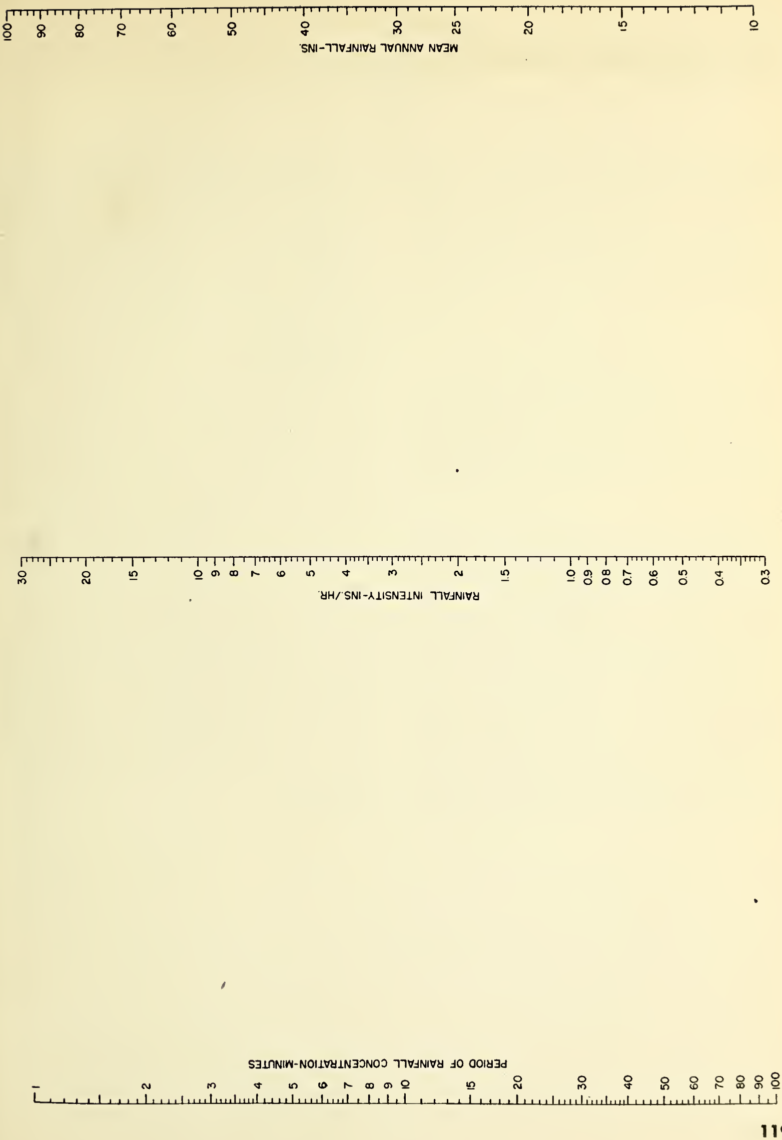
$$I = 0.3R/T^{0.5}$$

was found to fit fairly closely the various individual formulas.

Accordingly, the chart was set up on this basis. However, lack of adequate record data from desert areas subject to flash storms of great intensity prompts the suggestion that for such areas the chart cannot be taken as more than an indication. There are likely to be other localities where conditions may bring about intensities that do not follow any of the usual mathematical expressions, and there, too, the results of the chart cannot be given too much weight.

One other thing should be pointed out in connection with the chart. It is intended to apply only to normal storm intensities, where an occasional lack of capacity in drainage works is of little importance. If, on the other hand, the exceptional storm that brings greater intensities can cause damage of consequence, allowance must be made for the storm of such intensity as may occur once in 100, 1,000, or 10,000 years. Various authorities have set up bases for such allowance, but multiplying the result from this chart by the sixth root of the period in years will give close agreement with other methods of making this allowance.

**Procedure.** To use the chart, it is necessary only to establish a straight line through the scale points corresponding to the given values of *Period of Rainfall Concentration* and *Mean Annual Rainfall* on their respective scales. Then, at the scale point of the intersection of this line with the central *Rainfall Intensity* scale is read the desired value.


MEAN ANNUAL RAINFALL-INS.
RAINFALL INTENSITY-INS./HR.
PERIOD OF RAINFALL CONCENTRATION-MINUTES

# 51. Rainfall Runoff

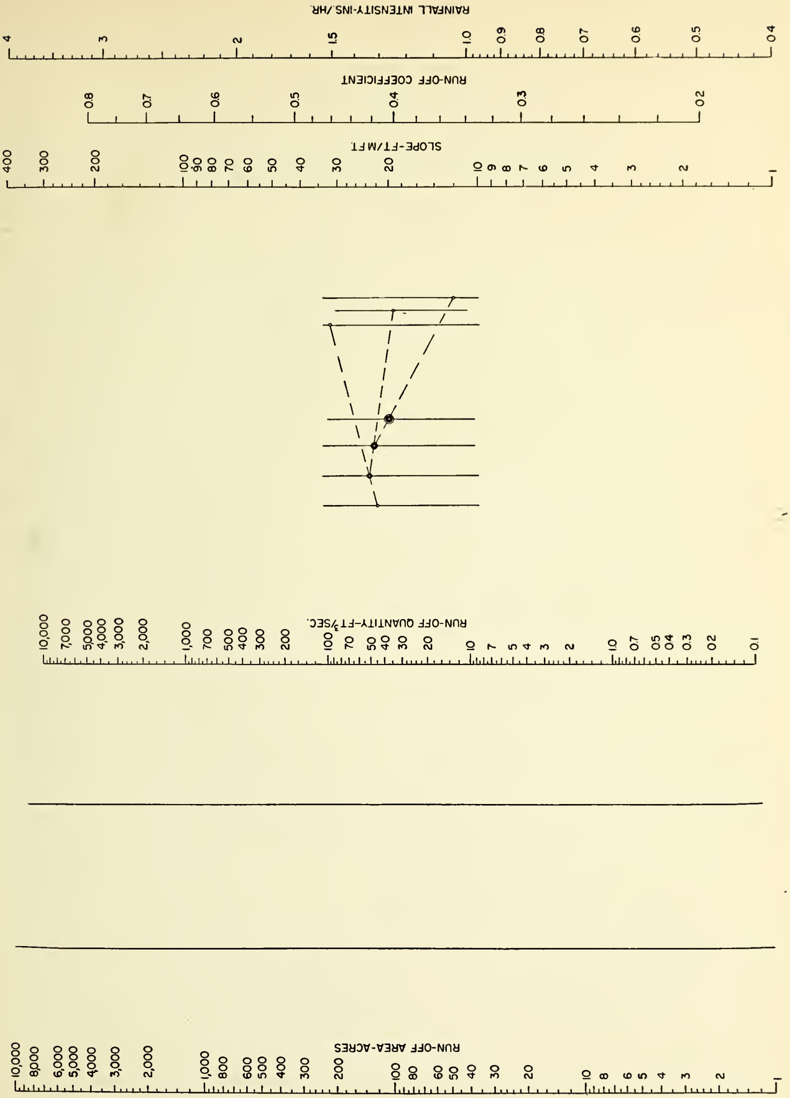
**Method for estimating** storm runoff, in order to establish a basis for the design of drainage works. It is based upon the widely used Burkli-Ziegeler formula, wherein the runoff flow, from a given area, in cubic feet per second is

$$Q = A^{0.75}S^{0.25}CR$$

where $A$ is the drainage area in acres, $S$ is the average slope to the outlet in feet per thousand, $C$ is a coefficient establishing the ratio between runoff and total precipitation, and $R$ is the rainfall intensity in inches per hour.

**Procedure.** A key diagram appears on the chart to indicate the sequence of operations in reaching the solution of a problem. Straight lines are established successively by scale points, and these lines intersect other scales to establish bases for later lines or the final result. First, a line is set up through the scale points for the given values of *Runoff Area* and *Slope,* on their respective scales, and the intersection of this line with the left-hand blank axis is noted. A second line is then established from this point to the selected value of *Runoff* coefficient on *that* scale, and the intersection of this line with the right-hand blank axis is noted. Then, a third line from this latter point to the assumed value of *Rainfall Intensity* on its scale will intersect the *Runoff Quantity* scale at the scale value which is the solution of the equation.

By way of illustration, assume a runoff area of 100 acres of an average slope of 50 feet per 1,000 feet, with a runoff coefficient of 0.30 under a rainfall intensity of 1.0 inch per hour. First a line through the scale points for 100 on the *Area* scale and 50.0 on the *Slope* scale sets a point on the left-hand blank axis. A second line through this point and the scale point for 0.30 on the *Coefficient* scale fixes a point on the right-hand blank axis. Third, a line from this point to the scale point for 1.00 on the *Intensity* scale intersects the *Runoff Quantity* scale at a scale point indicating 25.0 cubic feet per second (computed result 25.23).


RAINFALL INTENSITY-INS./HR.
RUN-OFF COEFFICIENT
SLOPE-FT/M FT
RUN-OFF QUANTITY-FT³/SEC.
RUN-OFF AREA-ACRES

# Group IV
# MECHANICS CHARTS

This group of charts is intended to cover a field of frequently recurring problems in machine design and simple structural design. The more involved problems of structures are generally solved by use of much more complex units than those included in the scope of these charts. But for analysis or design of such items as connecting links under compression (columns), machine-base frames, or other more or less simple beams, and other machine parts, this group of charts has proved to be of enormous advantage in saving both time and effort.

There is one chart covering columns within the range of compression members commonly found in mechanical design. It covers various types of end connections and different materials, as well as various common cross sections. Two charts have to do with beams, as to stresses and moments. They cover only a small part of the field of beams but *do* cover the part that brings the most frequent problems.

Two charts for helical springs, compression or tension, are included, each of them based upon a different approach to the problem, as dictated by the conditions obtaining. There are three charts having to do with belting one for determination of exact length and two for determining capacity or cross section, depending upon the approach to the problem. Then there is a chart covering a wide range of problems having to do with centrifugal force, in such form as to be useful in design of fastenings or supports for rotating parts.

Finally, there is included with this group a chart having to do with driven piling. True, this is only tenuously related to mechanics, but it is with this group because it has even less connection with any of the other groups.

Each of the charts is accompanied by explanatory text, and, in most cases, there is a key diagram showing operation of the chart, to save reference to the text every time the chart is used.

## Charts in Group IV

52. Column Loading
53. Maximum Beam Moment
54. Beam Stress
55. Compression Springs (I)
56. Compression Springs (II)
57. Belt Width (Light Loading)
58. Belt Design
59. Belt Length
60. Torsional Moment
61. Torsional Stress
62. Centrifugal Force
63. Pile Loading

# 52. Column Loading

**Method for the solution** of many of the problems involving simple columns, struts, links, etc., in compressive loading, that face the engineer in fields other than those where, columns being of primary importance, more complex forms may be dictated. The chart is based upon the Rankine formula, applying to columns having a slenderness ratio ($l/r$) of 20 to 120, the ratio between column length and the radius of gyration of the section.

The formula is widely used, but because of the form and the work involved in its solution, simpler formulas may often be more attractive. However, with this chart, the advantages of this formula may be had without the tedious computations being necessary. The formula is

$$P = \frac{SA}{1 + K(l^2/r^2)}$$

or, as really represented in the chart

$$\frac{P}{S} = \frac{A}{1 + K(l^2/r^2)}$$

where $P$ is the load on the column in pounds, $S$ is the maximum stress in the column in pounds per square inch, $A$ is the cross-sectional area of the column in square inches, $K$ is a coefficient depending upon column material and conditions of end supports, $l$ is the column length, and $r$ is the radius of gyration of the cross-sectional area, the two latter being in the same units.

**Procedure.** The chart is essentially in three sections. At the right-hand side there is an intersection-type-nomograph section which evaluates the terms having to do with column length, material, and manner of supporting the ends. In using the chart, an intersection is determined where the curved line for the given material and end-support condition intersects the vertical line corresponding to column length.

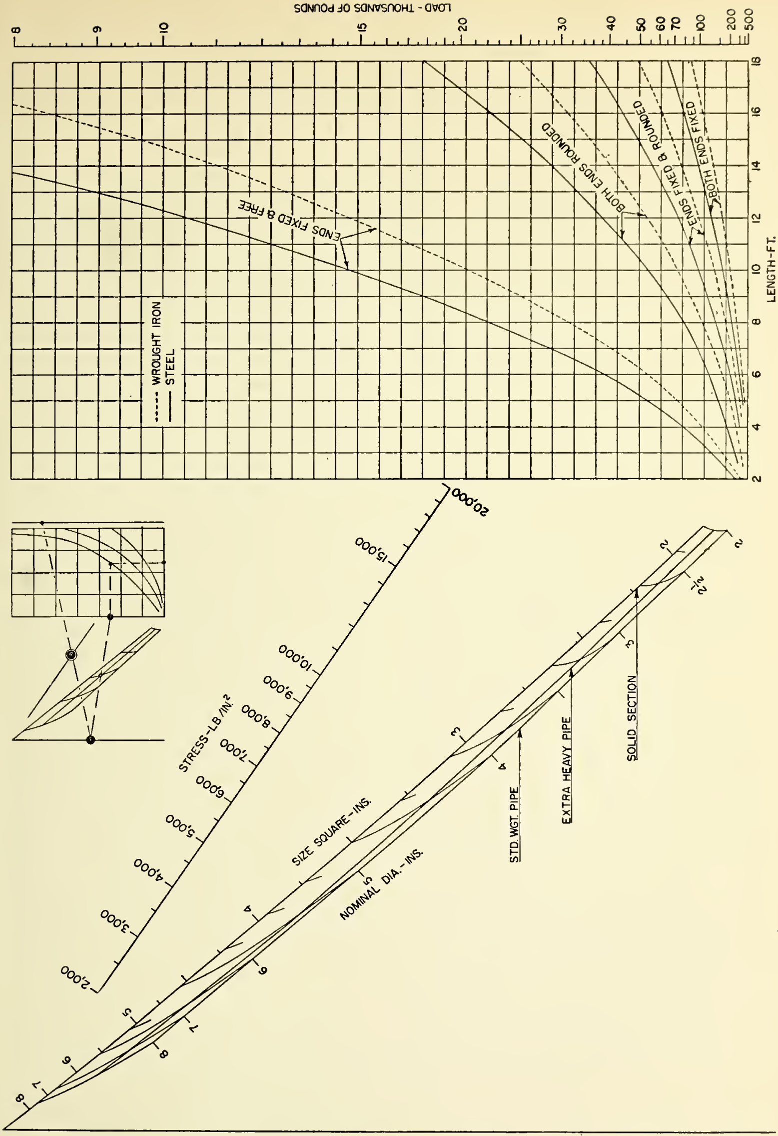
This vertical intercept is then carried horizontally across to the left-hand edge of this portion of the chart, which is one of the axes of the second section, an alignment nomograph. A straight line is established by this scale point through the scale point corresponding to the given conditions for column section, in the family of sloping lines and curves on the left-hand side of the chart. The upper scaling on the upper sloping line is for square sections, while the curves denote round sections, either solids or standard-weight or extra-heavy pipe. The line so established is carried over to the blank axis, and its intersection noted there.

The third section of the chart then consists of this scale and *Load* and *Stress* scales. To determine the unit stress, a straight line is established through the scale point for the total *Load* and the intersection previously noted on the blank axis. Then the unit *Stress* is read at the intersection of this line with the scale so designated.

To illustrate use of the chart, a solution will be carried out for a column of 4-inch extra-heavy steel pipe, with one end fixed and the other free, under a loading of 10,000 pounds. First, in the right-hand section of the chart, the intersection is deter-

mined for the vertical line of 10-foot *Length* with the curve for *Ends Fixed & Free—Steel.* This intercept is then carried horizontally to the left-hand edge of this section.

Then from this point a line is projected through the intersection of the curve for *Nominal Diameter* of 4, with the inner sloping line for *Extra Heavy Pipe,* and its intersection with the blank axis is noted. Then a line established by this point and the scale point for 10,000 pounds *Load* intersects the *Stress* scale at a scale point indicating 11,800 pounds per square inch (computed value 11,863).


LOAD - THOUSANDS OF POUNDS
LENGTH-FT.
WROUGHT IRON
STEEL
ENDS FIXED & FREE
BOTH ENDS ROUNDED
ENDS FIXED & ROUNDED
BOTH ENDS FIXED
STRESS-LB/IN.²
SIZE SQUARE-INS.
NOMINAL DIA.-INS.
STD. WGT. PIPE
EXTRA HEAVY PIPE
SOLID SECTION

# 53. Maximum Beam Moment

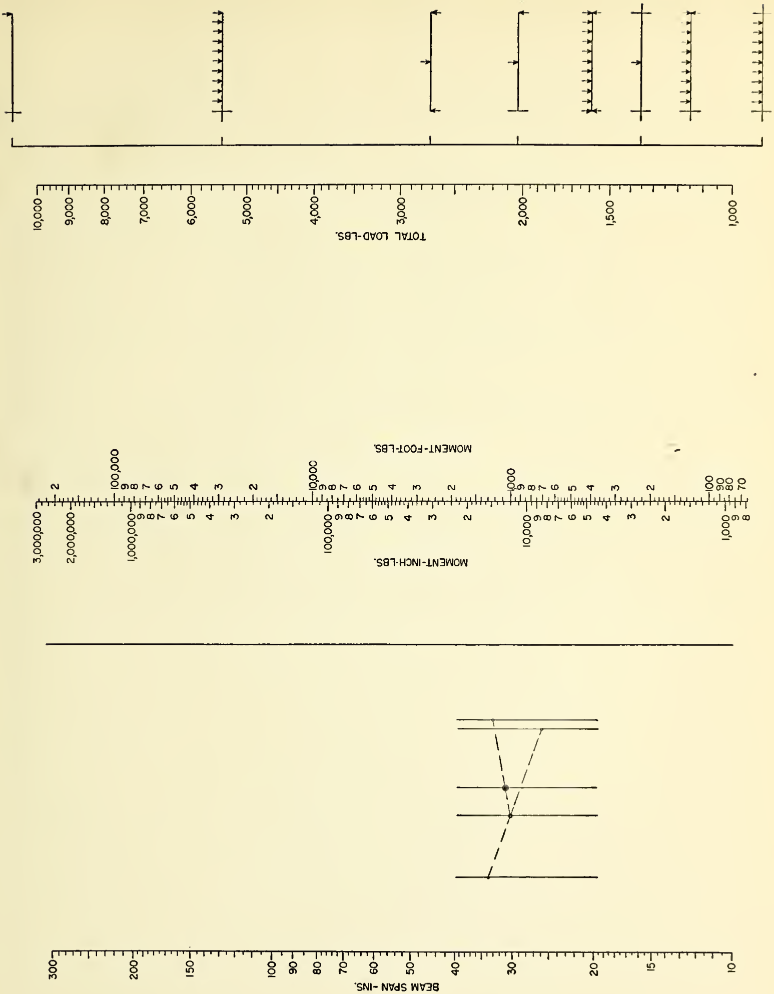
**Method for the determination** of the maximum-moment value in beams under various types and magnitudes of loadings and with various spans. The chart is based upon the conventional formula, wherein the maximum moment is equal to the product of total load and span length, together with a factor varying with the condition of support and loading.

In the chart, the conditions of support and loading are indicated on the appropriate scale by diagrams, which, it is hoped, may be self-explanatory. However, for further clarification, a few words might be in order. Uniform loading throughout the length of the beam is indicated by a series of uniformly spaced load arrows. Concentrated loads are represented by a single arrow at the point of loading, center or end, as the case may be. Simple unfixed supports are represented by arrows pointing upward, while fixed cantilever supports are represented by a continuation of the beam line through an end line. In each case, the diagram is related to a scale point on this axis corresponding to the value of the coefficient applying in the particular case.

**Procedure.** To use the chart, a straight line is established through the scale points for the *Total Load* and *Beam Span,* and the intersection of this line with the blank axis is noted. Then, a second straight line is established from this latter point to the scale point corresponding to the given conditions of loading and support. The intersection of the latter line with the *Moment* scale then gives the value of the maximum moment. This figure may be taken in foot-pounds or inch-pounds, as desired, but the original values must be in the units as noted on the original scales.

By way of illustrating use of the chart, assume a uniformly loaded beam, supported by simple uplift between supports 70 inches apart, the total load being 4,000 pounds. First, a straight line is established through the scale points for 70 on the *Beam Span* scale and 4,000 on the *Beam Load* scale, and the intersection of this line is noted on the blank axis.

Then, a second straight line is established through this point and the scale point corresponding to simple-support, uniformly loaded beams, the fifth diagram from the top (with a coefficient of 0.125). This newly established line intersects the *Moment* scale at a scale value indicating the moment as 35,000 inch-pounds (identical with numerical calculation).


TOTAL LOAD-LBS.
MOMENT-FOOT-LBS.
MOMENT-INCH-LBS.
BEAM SPAN - INS.

# 54. Beam Stress

**Method for determining** the relationship between stress, moment, and section modulus of simple beams in problems that frequently face the engineer in fields outside the purely structural field. In combination with the preceding chart, it is possible to work out the complete solution of many problems in simple beams. Chart 21, covering determinations of section moduli, will also find use with these two in such problems.

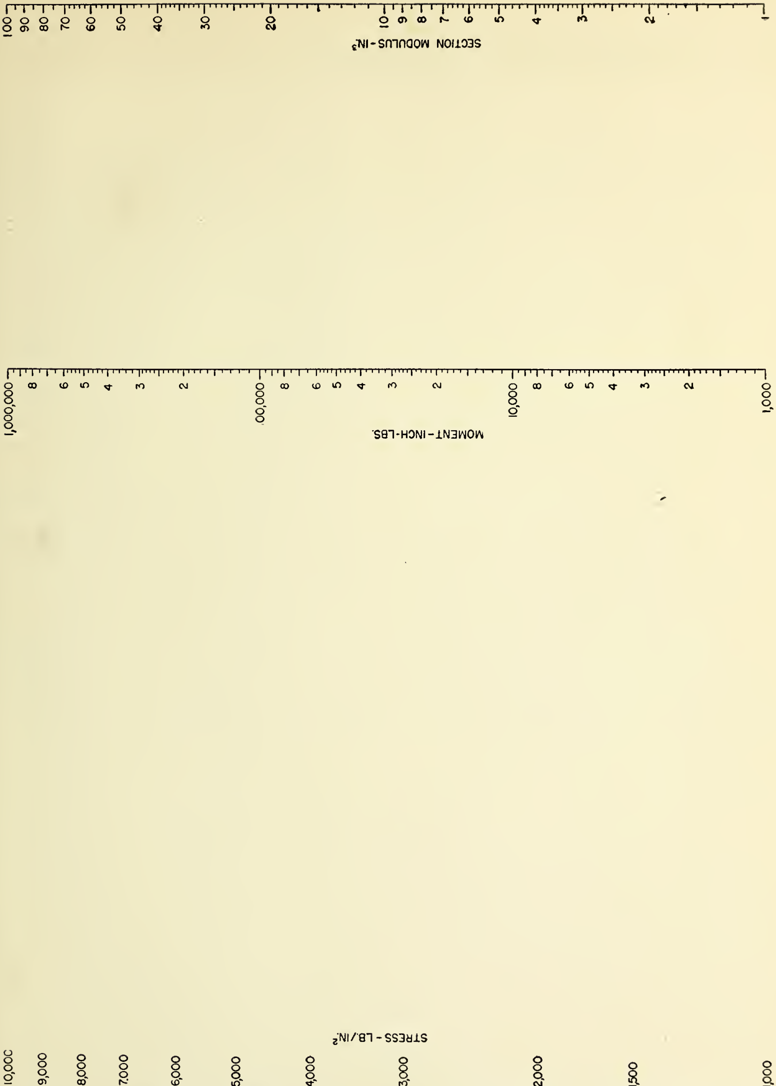
The chart is based upon the conventional formula for the relationship, which is

$$M/S = I/c$$

where $M$ is the moment in inch-pounds at the section in question, $S$ is the maximum unit stress in the section, being at the extreme outer fibers, and $I/c$ is the section modulus of the section, in inches cubed (equal to the section moment of inertia divided by the distance from neutral axis to outer fibers of the section).

**Procedure.** To use the chart, it is necessary only to establish a straight line through the scale points for any two of the scale quantities *Stress, Moment,* and *Section Modulus,* whereupon the third term is read at the scale point on that third axis where it is intersected by the line so established.

By way of illustration, assume a limiting stress of 2,000 pounds per square inch in a beam at a section where the moment is 250,000 inch-pounds. A line through the scale point of 2,000 on the *Stress* scale and 250,000 on the inch-pound scaling on the *Moment* scale, when projected, intersects the *Section Modulus* scale at a scale value of 125 inches cubed (the true value).


SECTION MODULUS - IN³
MOMENT - INCH-LBS.
STRESS - LB/IN²

# 55. Compression Springs (I)

**Method for the determination** of the characteristics of helical springs in either tension or compression. It will serve to arrive at designs of springs in many of the problems confronting an engineer. Naturally, in any case where designs must be to very close limits, or where large quantities may dictate designs to use the absolute minimum of material, the spring manufacturer should be called upon for design.

The formula upon which the chart is based is the conventional one, where the maximum safe load is

$$P = \pi D^3 S / 8d$$

where $D$ is the diameter of the wire used in the spring, in inches, $S$ is the maximum allowable stress in pounds per square inch, and $d$ is the mean diameter of the spring helix. Obviously, the stress limit can vary greatly, and in addition to variations with wire size and heat-treating, there is a rather wide variation in the recommendations as given by various manufacturers.

**Procedure.** The chart has been designed to take into consideration the average variation in stress limit, and results give a reasonable mean of the recommendations of various manufacturers. To use the chart, a straight line is established through the scale points corresponding to known or assumed values of any two of the three quantities *Wire Diameter, Mean Coil Diameter,* and *Safe Load.* The third figure is then read as the scale value at the intersection of this straight line with the third scale.

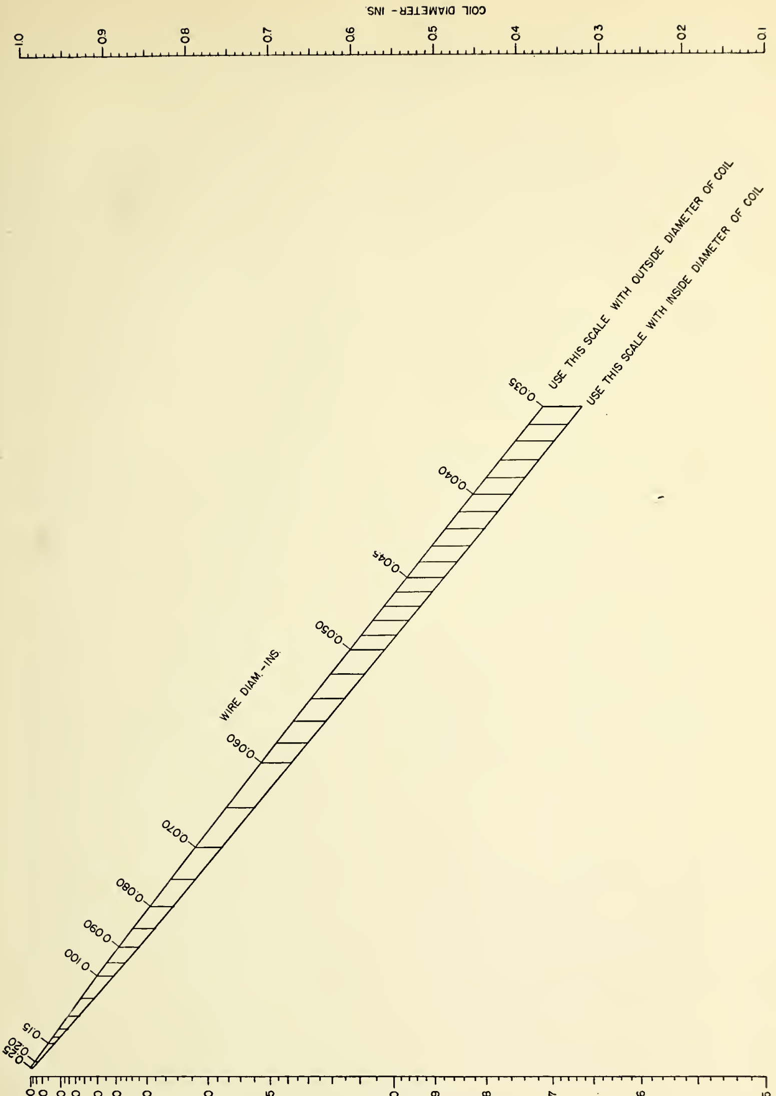
For instance, the safe load on a spring made of No. 9 wire (0.1483-inch diameter) with a mean coil diameter of 2.0 inches is determined by a straight line through 0.1483 on the *Wire Diameter* scale and 2.00 on the *Mean Coil Diameter* scale. This line intersects the *Safe Load* scale at a scale value of 58 pounds. A chart furnished by one of the leading spring manufacturers gives the safe load on such a spring as 64 pounds, but another shows only 39.


MEAN COIL DIAM.-INS.
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.5 2.0 2.5 3.0 4 5 6
SAFE LOAD-LBS.
10,000 7,000 5,000 3,000 2,000 1,000 500 300 200 100 70 50 30 20 10 7 5 3 2 1
WIRE DIAM.-INS.
1.5 1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.20 0.15 0.10 0.08 0.07 0.06 0.05 0.04 0.03 0.02 0.015

# 56. Compression Springs (II)

**Method for the design** of helical springs, both compression and tension, in a class of problems frequently met by the engineer. This chart is based upon the formula given in the discussion accompanying the preceding chart, with the same assumptions as to variation of limiting stresses. However, the chart design and scaling are arranged to use inside or outside diameter of the helical spring, rather than the mean diameter. This has been found convenient in the design of springs to fit inside a cylindrical bore, limiting outside diameter, or over a spindle or pin, fixing or limiting the inside diameter.

**Procedure.** To use this chart, a straight line is established between the scale points corresponding to *Coil Diameter* and *Load.* The intersection of this line then gives, at the scale point on one of the sloping lines, the (round) wire size required for the spring. If the diameter figure used was outside diameter, the intersection is noted on upper line, so marked. On the other hand, if inside diameter was used, the scale value is read at the intersection with the lower line, also so indicated.


COIL DIAMETER - INS.
USE THIS SCALE WITH OUTSIDE DIAMETER OF COIL
USE THIS SCALE WITH INSIDE DIAMETER OF COIL
WIRE DIAM.-INS.
SAFE LOAD-LBS.

# 57. Belt Width (Light Loading)

**Method for the determination** of the required width of leather belting to transmit power under certain conditions, with varying power, speed, and pulley diameter. This chart, the first of two having to do with the same subject, should not be used where belt speeds much exceed 2,000 feet per minute, where angle of arc of contact deviates much from 180°, or in drives of such size as to make close design of economic importance.

The chart is based upon a relationship that is a compromise between various charts, tables, and special slide rules, distributed by manufacturers of power-transmission equipment, and gives belt width in inches (for a single belt) as

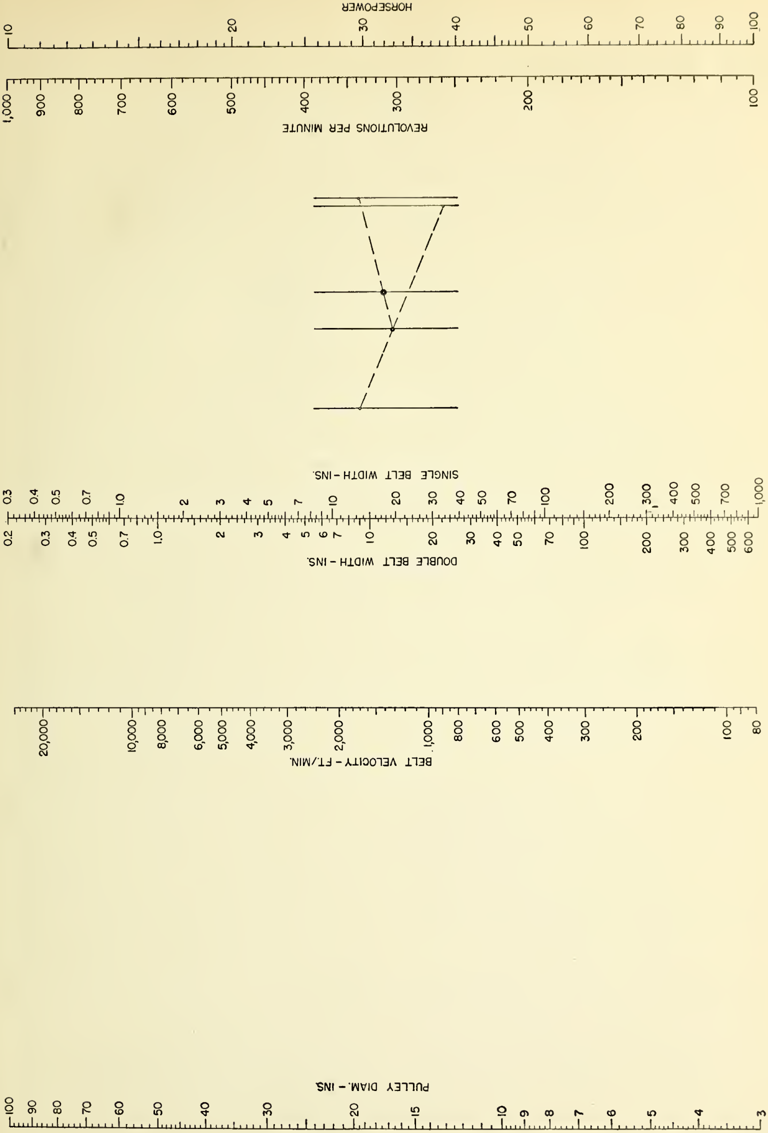
$$W = 3{,}000 \text{ Hp}/DN$$

where Hp is the horsepower to be transmitted, $D$ is the pulley diameter in inches, and $N$ is the speed of the smaller pulley in revolutions per minute. One portion of the chart also gives a determination of belt speed in feet per minute corresponding to $V = \pi DN/12$, or $0.2618DN$.

**Procedure.** To use the chart, a straight line is established through the scale points corresponding to the given values on the *Pulley Diameter* and *Revolutions per Minute* scales, and the scale point is noted on the *Belt Velocity* scale where this line intersects. A second line is established through this point and the scale point for the *Horsepower,* and the required *Belt Width* is read as the scale point where the line intersects this scale, which is scaled double, for single belts on one side and for double belts on the other. It may be noted that the double-belt scaling is two-thirds that of the single-belt figures, corresponding to a constant factor of 2,000 instead of 3,000 in the original equation.

Use of the chart is illustrated by determining the required belt width for a drive to carry 10 horsepower, with a 10-inch pulley at 500 revolutions per minute. First, a line is established through 10.0 on the *Horsepower* scale and 500 on the *Revolutions per Minute* scale. This intersects the *Belt Velocity* scale at a scale point corresponding to 1,310 feet per minute (true computed value 1,309).

Then a second line is established through this point and the scale point for 10.0 on the *Horsepower* scale, and its intersection with the *Belt Width* scale is at a scale point of 5.9 on the *Single Belt* scale (3.93 on the *Double Belt* scale). A computation on the basis of the given equation results in a figure of 6.00 as the single-belt width, and the average of tabulations and charts that have come to the attention of this author result in an average of 6.10 inches.


HORSEPOWER
REVOLUTIONS PER MINUTE
SINGLE BELT WIDTH - INS.
DOUBLE BELT WIDTH - INS.
BELT VELOCITY - FT./MIN.
PULLEY DIAM. - INS.

# 58. Belt Design

**Method for the determination** of the width of leather belting required to carry a specified load. This chart is the second of two on this subject; the first, Chart 57, is based upon use of approximate methods, suitable for use only with comparatively low belt speeds. As such speeds are increased, the centrifugal effect on the belt in passing around pulleys adds an artificial tension as well as having an effect upon the coefficient of friction between belt and pully, and as the speed increases, these factors have an increasing effect upon the design. In the earlier chart, the equation used made no provision for them, and, as stated there, it should not be used where belt speeds are much in excess of 2,000 feet per minute.

Design computation by means of the exact mathematical formulas, a series of five, is an involved, tedious process. This chart provides a solution based upon exact methods, but requiring only a few seconds to arrive at a solution. The equations used as a basis, evaluating the various tension components, the variations in friction coefficient, and resultant unit capacity of the belting, may be found in treatises on mechanical transmission of power and in handbooks, so there is no need to take the space required to repeat them here.

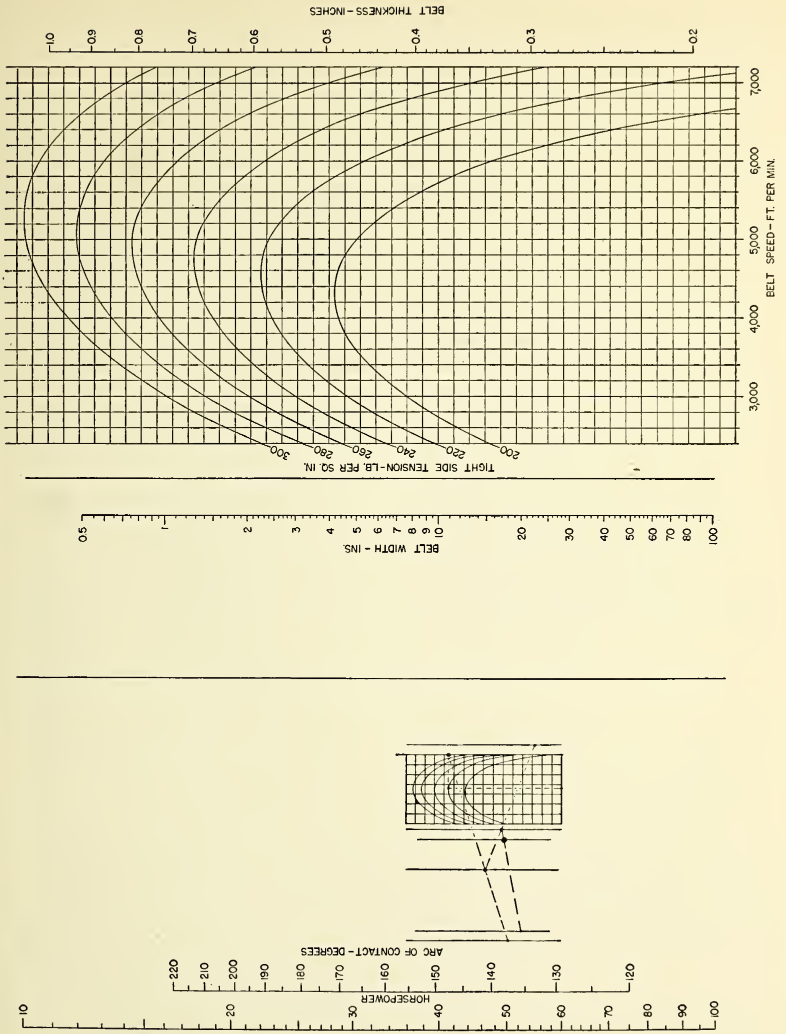
**Procedure.** The chart incorporates an intersection-type portion and then a three-stage alignment section, and in using it, a point is first found in the intersection portion, where the vertical line corresponding to *Belt Speed* intersects the curve for the assumed maximum *Tight Side Tension.* The former may be determined easily from Chart 57, which is the reason that parts of that chart were carried beyond the limits of practical use of that chart for complete solution.

As to the figure selected for *Tight Side Tension,* 240 pounds per square inch is generally recommended, although it may be as high as 300 for new belts of good quality or as low as 200. Belt-design treatises by Barth and Sellers represent values of 200 and 255 pounds per square inch, respectively.

The next step in the use of the chart is to extend the intercept found as above. It is carried over horizontally to the heavy line near the right-hand edge of this portion of the chart. This heavy line, used as an axis in the alignment section, is that corresponding to the vertical line for a belt speed of 7,000 feet per minute.

A straight line is then established through this point and the scale point corresponding to the power to be transmitted, on the *Horsepower* scale, and the intersection of this line with the first blank axis (the one of the two that is farther toward the left) is noted. A second straight line is established through this line and the scale point for the assumed value of *Belt Thickness* on the scale so designated, and the intersection of this line is noted on the second blank axis.

Then, a third straight line is established through this point and the computed value for the *Arc of Contact* on the smaller of the pulleys in the drive, and at the inter-


BELT THICKNESS - INCHES
BELT SPEED - FT. PER MIN.
TIGHT SIDE TENSION-LB. PER SQ. IN.
BELT WIDTH - INS.
ARC OF CONTACT - DEGREES
HORSEPOWER

section of this line with the scale of *Belt Width* the desired scale reading is read directly. It is felt that the scalings of the various terms will cover all conditions which may be met in practice, with the exception of horsepower. For values of horsepower outside the scale limits, the actual horsepower may be multiplied by 10 or 0.1 to arrive at a figure within the limits, afterward multiplying the final determination by 0.1 or 10, respectively, to obtain the true result.

It might be mentioned that for 180° arc of contact, and with belt speeds under 3,000 feet per minute, results from this chart do not deviate greatly from those taken from Chart 57.

By way of illustrating use of the chart, assume conditions requiring the transmission of 75 horsepower, with a single-belt speed of 4,800 feet per minute, tight-side belt tension being limited to 240 pounds per square inch, the arc of contact being 180°. First, the intersection of the vertical line of 4,800 feet per minute *Belt Speed* and the curve for 240 pounds per square inch *Tight Side Tension* is noted in the intersection-nomograph section, and its intercept is carried over to the main axis of this portion, on the vertical line coinciding with the 7,000 feet per minute belt speed.

Then a line is fixed through this point and the scale point for 75 on the *Horsepower* scale, and the intersection of this line with the first blank axis is noted. Then another line is laid from this point to the assumed value of *Belt Thickness,* taken as 0.25 for the single belt, and the intersection of this line is noted on the second blank axis.

Then another line is set from this point to the assumed value on the *Arc of Contact* scale, and the intersection of this line is noted on the *Belt Width* scale as 16.3 inches. (The result achieved by carrying out full computation is 16.41.) By way of emphasizing the limitations of Chart 57, it may be noted that the result by use of *that* chart would be 10.8 inches. This illustrates forcibly the effect of the factors of centrifugal-tension component and varying friction coefficient at high belt speeds.

# 59. Belt Length

**Method for the determination** of the length of an open belt transmitting power between two pulleys and for finding pulley diameters which, at given center distance, will give varying drive-speed ratios with given belt length. This latter provision is of assistance in designing varying ratio drives with step-cone pulleys.

This is one of the cases mentioned in the foreword to this volume where the intersection type of nomograph seems better adapted to the problem than the alignment type. It is true that there have appeared alignment nomographs for the solution of the problem, but they seem to be based upon approximate rather than exact formulas. It may be seen that in the chart the lines drawn on the basic system of rectangular coordinates are not straight, but slightly curved, and it is this deviation that marks the difference between the exact and approximate methods of solution.

The formula used in designing this chart is the exact one, where the belt length is

$$L = 2\sqrt{C^2 - (R_1 - R_2)^2} + \pi(R_1 + R_2) + 2(R_1 - R_2)\sin^{-1}\frac{(R_1 - R_2)}{C}$$

where $C$ is the distance between centers of the pulleys and $R_1$ and $R_2$ are the radii of the two pulleys, all dimensions being in the same units. Computation of belt lengths by the formula is, at best, a rather tedious process, but if belt length and center distance are the known quantities, computation of pulley diameters to fit is a long process of trial and error, avoided by the use of this chart.

**Procedure.** To simplify the chart, all dimensions are reduced to ratios to the distance between centers, as unity. Thus, the first step in using the chart is to divide the pulley diameters by the center distance, if it is belt length that must be determined. If center distance and belt length are the fixed quantities, then the belt length is divided by the center distance to obtain its ratio to center distance.

Then, using the chart to find belt length, a point is located on the chart at the intersection of the horizontal line scaled at the value of one pulley diameter and the sloping (slightly curved and generally parallel) line scaled at the value of the other pulley diameter (both as ratios to center distance), interpolating as necessary. The belt length, as a multiple of center distance, is then read by interpolating between the vertical lines, scaled at the bottom.

It will be noted that there are also radial lines, slightly curved, and these are lines scaled to denote the speed ratio between the shaft pulleys. Thus, if but one pulley diameter is known, for a fixed ratio between shaft speeds, the intersection to be found is that of the line of known diameter with the line of the fixed ratio, whereupon the diameter of the other pulley and the length of belt can be read off, interpolating as necessary.

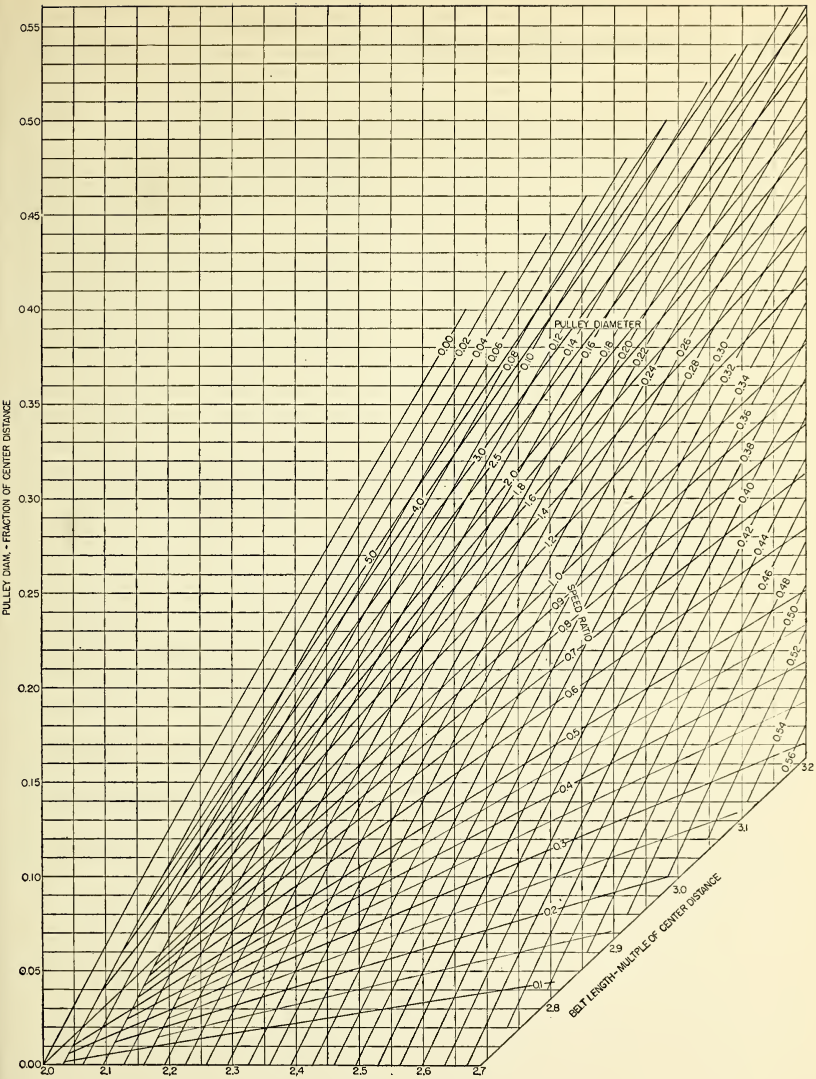
The third use of the chart is when, with center distance and belt length fixed, pulley diameters for other ratios are required. In this case, the intersection first deter-

mined is that of the vertical line corresponding to belt length, with the radial line of desired speed ratio. Then, interpolating as necessary, the two pulley diameters are read from the horizontal and sloping lines, respectively.

To illustrate use of the chart, a case is assumed with a distance between centers of 6 feet, or 72 inches. Speed ratios of 1:1, 1:1.5, and 1:2 are to be provided, the pulleys being 10 inches in diameter for the first case.

The 10-inch diameter is 0.1389, as a multiple of the center distance, so a point of intersection is found where, interpolating as necessary, this value in one set of *Pulley Diameter* lines intersects either the same value in the other set of *Pulley Diameter* lines, or the radial line for 1:1 ratio. This intersection gives the value of *Belt Length* as 2.436. Simple multiplication (by true center distance) then gives the actual belt length as 175.39 inches.

Belt length now being fixed, the intersection of the line for this value is then noted with the lines of 1:1.5 and 1:2 *Speed Ratio* to give the diameters as 0.1105 and 0.1660 for the 1:1.5 ratio and 0.0920 and 0.1840 for the other. Again simple multiplication gives the true diameters as 7.96 and 11.95 inches for the 1:1.5 ratio and 6.62 and 13.25 inches for the other. Actual computations gave results of 175.42 inches belt length for the 1:1 ratio, 175.32 for the 1:1.5 ratio, and 175.45 for the 1:2 ratio.


PULLEY DIAM. - FRACTION OF CENTER DISTANCE
PULLEY DIAMETER
SPEED RATIO
BELT LENGTH - MULTPLE OF CENTER DISTANCE
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
2.0
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
3.0
3.1
3.2

# 60. Torsional Moment

**Method for the determination** of the torsional moment induced in a torsional member subjected to a given horsepower at a known speed. Tables and charts are available for finding the shaft size required to transmit power under conditions involving nothing but torsion, but some problems require analysis of more than mere torsion, and this chart has been found useful in those cases. In addition, it is useful even in those cases in which torsion is the only problem, in which case, with the following chart, a complete solution is possible.

The formula upon which the chart is based is the conventional one, quoted directly as given, or given in such form as to be easily resolved, wherein the torsional moment in inch-pounds is

$$M = 63{,}023 \text{ Hp}/N$$

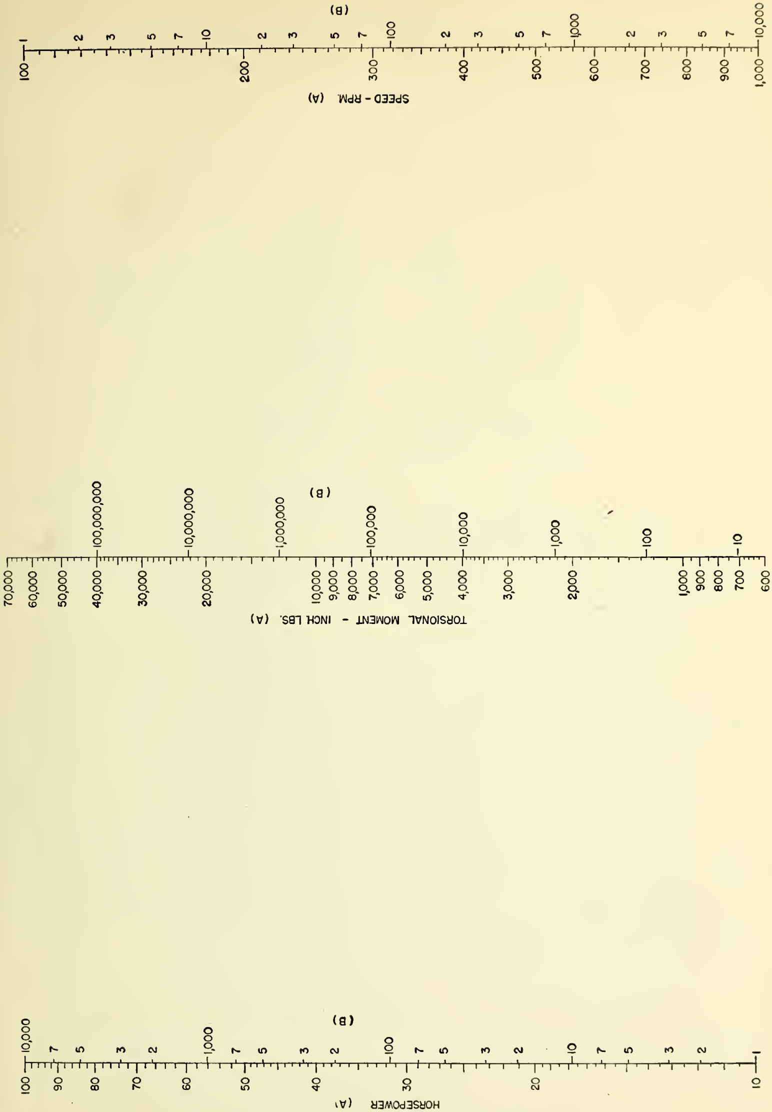
where Hp is the horsepower transmitted and $N$ is the shaft speed in revolutions per minute.

**Procedure.** To use the chart, it is necessary only to establish a straight line through the scale points on the *Horsepower* and *Speed* scales, and then read the scale point at the intersection of this line with the scale for *Moment*. All three scales are double-scaled, the *A* scale for accurate determinations, and the *B* scale for determination of the general range of the final result.

By way of illustration of use of the chart, assume the transmission of 60.0 horsepower at 300 revolutions per minute. First, to determine the general range of the result, use the *B* scales, and establish a line through the scale point (even approximately) for 60 on the *Horsepower* scale and that for 300 on the *Speed* scale. The intersection of this line with the *Moment* scale indicates that the moment is in the range between 10,000 and 100,000 inch-pounds.

Then, using the *A* scalings, a line is established through the scale points for 60 on the *Horsepower* scale and 300 on the *Speed* scale, which is found to intersect the *Moment* scale at a scale point indicating 12,600 inch-pounds (computed value being 12,605).

Had the horsepower value been 600, the first step would have indicated the true figure to be between 100,000 and 1,000,000, so instead of taking the direct reading, the same integers would have been used, but multiplied by 10, for a final result of 126,-000 inch-pounds.


SPEED - RPM (A)
(B)
TORSIONAL MOMENT - INCH LBS. (A)
(B)
HORSEPOWER (A)
(B)

# 61. Torsional Stress

**Method for the determination** of the torsional stress in a shaft under a known moment, for a given diameter, or for determining the diameter required with a given moment and stress limitation. This chart, in combination with Chart 60, will then solve problems wherein torsion alone is of importance, and with a minimum of effort.

The equation upon which the chart is based is the usual one for the problem, wherein the moment, or torque, exerted by the shaft is

$$M = \frac{\pi d^3 S}{16} = 0.19635 d^3 S$$

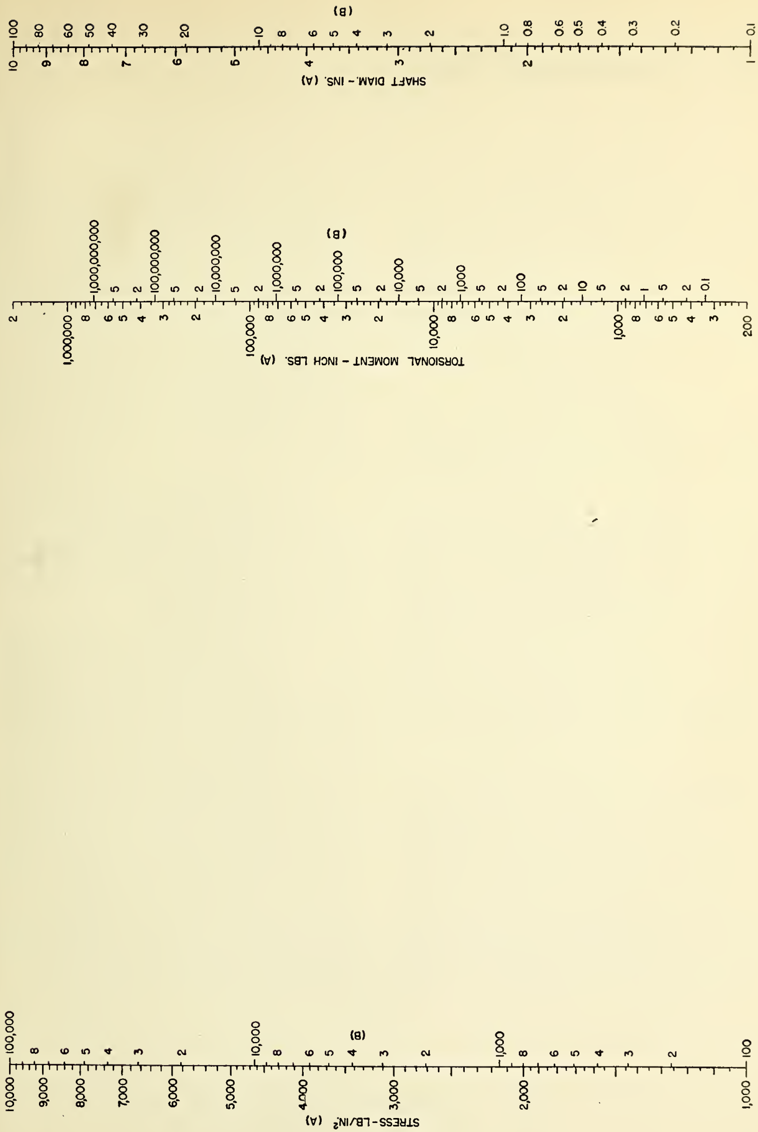
where $d$ is the shaft diameter in inches and $S$ is the torsional stress in pounds per square inch. The scales on the chart are double-scaled, the $A$ scales for accurate evaluation, and the $B$ scales for finding the general range of the result.

**Procedure.** To use the chart, recourse is first taken to the $B$ scalings, after which the $A$ scalings are used to determine the significant figures. In each case, a straight line is established through the scale points for any two of the three variables *Stress, Moment,* and *Shaft Diameter,* and at the intersection of this line with the third scale is read the third quantity.

By way of illustration we may determine the moment exerted by a shaft 3.50 inches in diameter under a maximum stress of 8,000 pounds per square inch. First, using the $B$ scalings, fix a line through scale points approximating 3.5 on the *Diameter* scale and 8,000 on the *Stress* scale, and the intersection of this line with the *Moment* scale indicates the moment to be between 10,000 and 100,000. Then, using the $A$ scales similarly, the intersection is at a scale point of 67,500 (computed result being 67,348).

If the first stage, by use of the $B$ scalings, had indicated a result between 1,000 and 10,000 (with different basic figures, of course), then, had the final scale reading been the same as above, the true value would have been 6,750.

Illustrating another sequence in the use of the chart, a case may be assumed requiring the design of a shaft wherein the torsional moment is to be 60,000 inch-pounds with a limiting stress of 7,500 pounds per square inch. In this case, then, a straight line is fixed through the scale points for 7,500 on the *Stress* scale and 60,000 on the *Moment* scale. This line is found, when prolonged, to intersect the *Diameter* scale at a scale point (interpolating) of 3.43 inches diameter (computed value being 3.446).


SHAFT DIAM. - INS. (A)
(B)
TORSIONAL MOMENT - INCH LBS. (A)
(B)
STRESS-LB/IN² (A)
(B)

# 62. Centrifugal Force

**Method for the determination** of the centrifugal effect resulting from rotation at various speeds of objects of various weights at various radii. The formula upon which the chart is based is the usual expression for this force, although here reduced to simple form, where the centrifugal force is

$$F = 0.00034N^2RW$$

where $N$ is the rotational speed in revolutions per minute, $R$ is the radius of the center of gravity of the rotating body, in feet, and $W$ is the weight of the rotating body, in pounds.

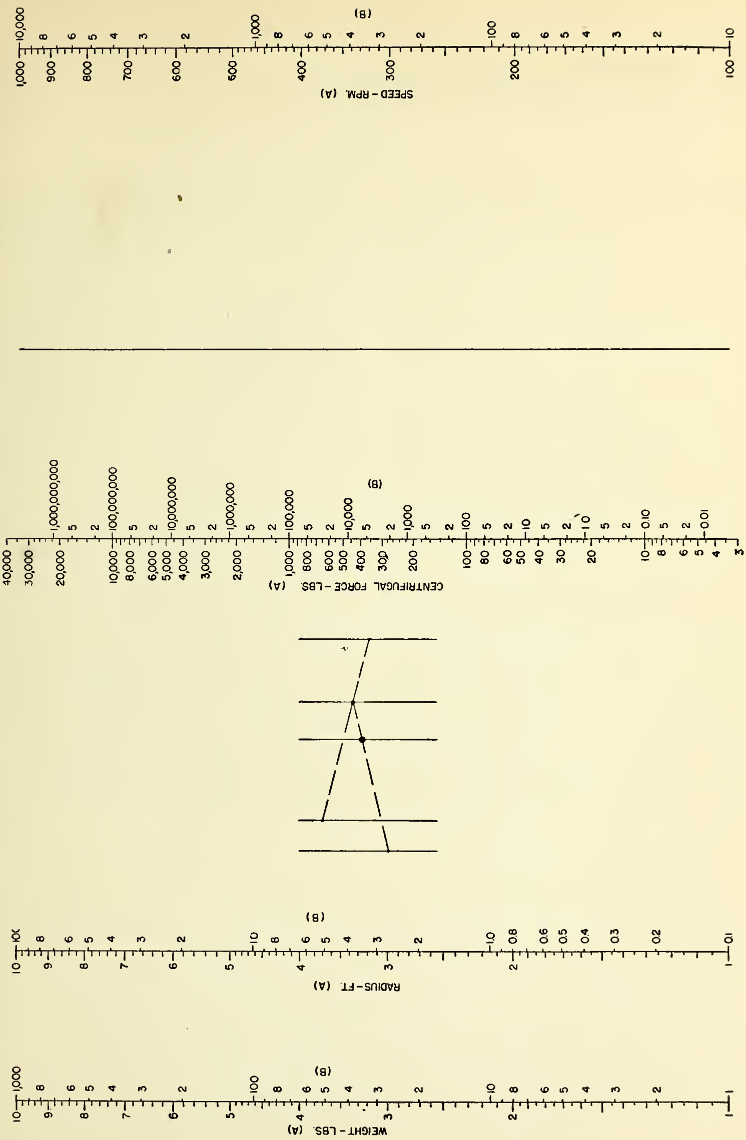
Many problems in this field will fall outside the primary scalings on the chart, those designated *A*. For this reason, a secondary scaling *B* appears, by which the approximate result is determined, at least to the extent of learning the number of figures before the decimal point. Then, recourse to scalings A, without regard for the decimal point, will give the figure itself.

**Procedure.** To use the chart, the first stage is in establishing a straight line through the scale points corresponding to the given values of *Speed* and *Radius* on their respective scales. The intersection of this line with the blank axis is then noted. A second straight line is then fixed through this point and the scale value of *Weight* on that scale, and at the intersection of this line with the *Centrifugal Force* scale is read the desired figure. The same process is followed with *A* or *B* scalings.

By way of illustration, a solution is to be found for a weight of 0.6 pound, rotating at a speed of 3,500 revolutions per minute at a radius of 0.35 foot. First, a line is set through the scale points corresponding to 3,500 on the *B Speed* scale and 0.35 (below 1.0) on the *B Radius* scale. The intersection of this line with the blank axis is then noted.

A second straight line is then established through this point and the scale point for 0.6 on the *B Weight* scale, and at the intersection of this line with the *Centrifugal Force B* scale, the result is noted as being between 500 and 1,000.

Now the same process is repeated using the *A* scales without regard for decimal points, using scale points for the basic data multiplied by such powers of 10 as bring them within the scale limits. Thus, the *Speed* scale point used will be 350, the *Radius* scale point will be 3.5, and the *Weight* scale point will be 6.0, and the final result is read as 875. By coincidence, the multiplications by powers of 10 cancel out in this instance, because the preliminary *B* operation indicated the result to be between 500 and 1,000, so the indicated result is the desired figure. Computation gives the exact figure as 874.65.


SPEED - RPM. (A)
(B)
CENTRIFUGAL FORCE - LBS. (A)
(B)
RADIUS-FT. (A)
(B)
WEIGHT - LBS. (A)
(B)

# 63. Pile Loading

**Method for the determination** of the safe load on piles, where the penetration at the last blow of the driver is taken as the criterion. The formula upon which the chart is based is one for which credit is given in handbooks to *Engineering News* and is in the form where the maximum safe load is

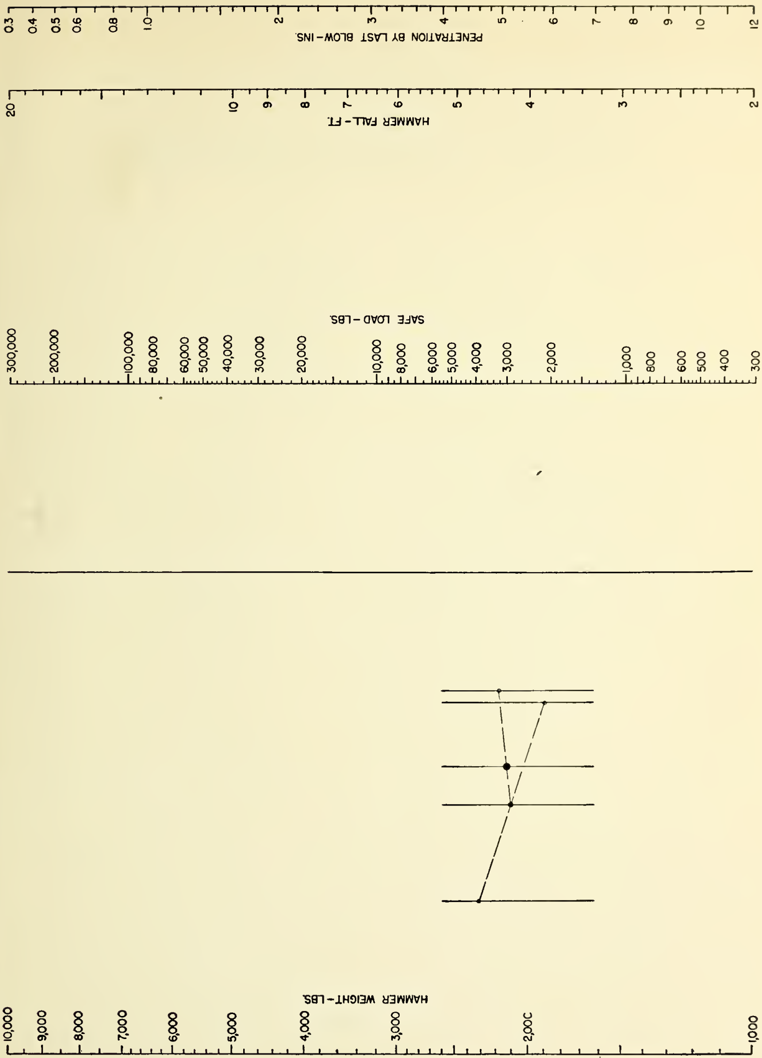
$$L = 2WH/(s + 1)$$

where $W$ is the total weight of the falling parts of the hammer assembly, $H$ is the height in feet of the free fall of the hammer assembly, and $s$ is the penetration of the pile under the influence of the hammer on the last drop, in inches.

**Procedure.** To use the chart, the first stage is the establishment of a straight line through the scale points corresponding to the known values on the *Hammer Weight* and *Hammer Fall* scales, noting the intersection of this line with the blank axis. A second straight line is then fixed by this point and the scale point corresponding to the known value of *Penetration by Last Blow* on the scale so designated, and at the intersection of this line with the *Safe Load* scale is read the scale value of the figure sought.

Where the known quantities, except penetration, lie beyond the limits of the chart, solution may be obtained by multiplying the quantities by powers of 10 such as will result in figures within the scale limitations, afterward multiplying the apparent result by the reciprocals of such factors to obtain a true solution.

By way of illustration, assume a weight of hammer (total falling parts) of 3,500 pounds, a drop of 6.0 feet, and a last-blow penetration of 8.0 inches. A line through scale points for 3,500 on the *Hammer Weight* scale and for 6.0 on the *Hammer Fall* scale is set, and its intersection is noted on the blank axis. A second line through this point and the scale point for 8.0 on the *Penetration* scale is then found to intersect the *Safe Load* scale at a scale point indicating a safe load of 4,650 pounds (computed value 4,667).


PENETRATION BY LAST BLOW - INS.
HAMMER FALL - FT.
SAFE LOAD - LBS.
HAMMER WEIGHT - LBS.

# Group V

# THERMODYNAMICS CHARTS

This group of charts has to do almost entirely with steam, although one or two of the individual charts find use in other fields. The first one of the group falls into this category, for while it is useful in plotting steam expansion curves, it is equally useful in plotting expansion or compression of other gases.

Most of the charts in this group have previously appeared in *Power* and other engineering publications, some as long as 20 years ago. But they have been found useful throughout the years and are presented again in this volume.

The first charts of this group are of more or less general nature in the field of steam, having to do with expansion, flow through piping and orifices, and calorimeter determinations of quality. The remainder of the group, however, have to do with boiler performance and tests. Of these, several cover fields of determination of specific losses in boilers, the last one being for calculation of over-all efficiency.

Although there is an explanation accompanying each chart, detailing methods of use and illustrations, most of the charts also bear a key diagram outlining the sequence of operations. These will very often save rereading the text at each use.

## Charts in Group V

# 64. Gas Expansion

**Method for making calculations** necessary for plotting gas expansion or compression curves, or for making other computations for determination of relationships between pressure and volume of a fixed amount of gas. The equation upon which the chart is based is the conventional one, where

$$PV^n = C$$

where $P$ is pressure, $V$ is volume, $n$ is an exponent, and $C$ is a figure that remains constant through any one series of computations.

Any units may be used for pressure and volume, but in any series of computations they must be consistent.

In arithmetical computation, the first step must be to set up the value of the constant upon the basis of original pressure and volume conditions and an assumed exponent. Then pressures may be computed for varying volumes, or volumes for varying pressures. Where the exponent is 1.0, which can happen only in theory, the arithmetical problem is simple. But the exponents applying to various gaseous materials under actual conditions are usually odd ones, making computation difficult and tedious. With the chart, however, it is a simple process.

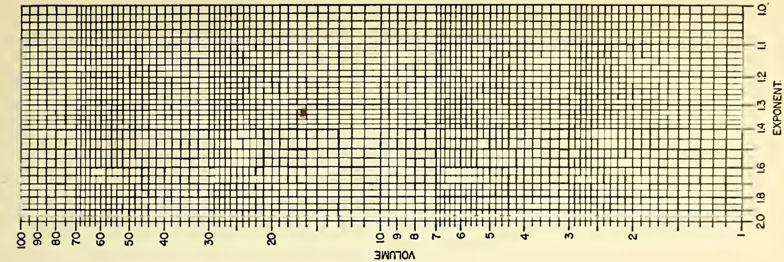
**Procedure.** First, the value of the exponent is set, and then the vertical line scaled for this value on the chart is used throughout computations for the particular gas and general range of operations. The second step is then to note on this vertical line the intersection of the horizontal line scaled for the initial value of volume, interpolating as may be necessary. Then a straight line is established through this point and the scale point corresponding to the initial *Absolute Pressure* on the scale so designated.

The intersection of this line is then noted on the blank axis, and this point is used as a basis for further steps. Thus, to determine the new volume at some different pressure, a straight line is established through this point and the scale point for the new value on the *Absolute Pressure* scale. Then, on the vertical line corresponding to the assumed exponent value, at the intersection of the line just established, may be read the new value of volume.

By way of illustration, data may be obtained for plotting the expansion curve of a volume of gas beginning with an absolute pressure of 300 pounds per square inch and occupying a volume of 5.00 cubic feet, the exponent being set at 1.33. The first step, of course, is as described above, locating the vertical line corresponding to the exponent, interpolating as necessary.

Next, a straight line is established through the scale point *on this line,* corresponding to the *Volume,* 5.00, and the scale point for 300 on the *Absolute Pressure* scale. The intersection of the line so established with the central, blank axis is then noted and is used in subsequent operations in this particular problem.

Now, the line is rotated about this point on the blank axis to find points on the


EXPONENT
1.0 1.1 1.2 1.3 1.4 1.6 1.8 2.0
VOLUME
100 90 80 70 60 50 40 30 20 10 9 8 7 6 5 4 3 2 1


ABSOLUTE PRESSURE
300 200 100 90 80 70 60 50 40 30 20 10

expansion curve, and with one end fixed at any selected scale point on the *Absolute Pressure* scale, the corresponding *Volume* is read as the intercept of the horizontal scale lines on the vertical line of assumed exponent at the point of intersection of the rotating line. In this case, reading at absolute pressures of 150, 80, and 40, the corresponding volumes are 8.35, 13.2, and 22.4, where computation gave 8.39, 13.20, and 22.75.

It should be emphasized that operation of this chart differs from others in this collection in that the intercepts on the vertical lines are *not* carried to a fixed axis associated with the graph section.

# 65. Orifice Steam Flow

**Method for the determination** of the flow of steam through rounded orifices. Accurate formulas for this problem are complex in form and tedious in use, so recourse is often had to approximate formulas, with consequent loss of accuracy, often to a large degree. This chart, however, combines the accuracy of an accepted formula with speed and ease of computation.

The formula upon which the chart is based is that due to Grashof, wherein the weight of steam flowing through a rounded orifice, in pounds per second, is

$$W = kap^{0.97}/62.5c$$

where $k$ is a factor depending upon the ratio of back pressure to initial pressure, equal to 1.00 when this ratio is less than 0.58, $a$ is the orifice area in square inches, $p$ is the initial pressure in pounds per square inch *absolute,* and $c$ is a factor depending upon steam quality or superheat. For wet steam, this latter factor is equal to the square root of the steam quality expressed as a decimal. For superheated steam, the factor is equal to $1.0 + 0.00065D$, where $D$ is the superheat in degrees Fahrenheit.

**Procedure.** The chart, as may be seen, is a combination of intersection- and alignment-type nomographs. In the intersection portion, at the right a family of curves is plotted, the vertical lines of the rectangular coordinate system being scaled for initial pressures, with curves scaled for back pressures. By way of clarification, it might be said that the 10-pound back-pressure curve continues upward to the right, being joined by the curves of all higher back pressures.

It is in this portion of the chart that any solution must start, in locating the intersection of the vertical line corresponding to the given condition of *Initial Pressure* (absolute), with the curve, or its extension in the sloping line, of the value of *Back Pressure.* Then, from this point of intersection, the same intercept, or vertical position, is found on the line forming the extreme right-hand edge of the intersection-type portion of the chart (being the vertical line corresponding to a pressure of 500 pounds), by following horizontally across the coordinate system.

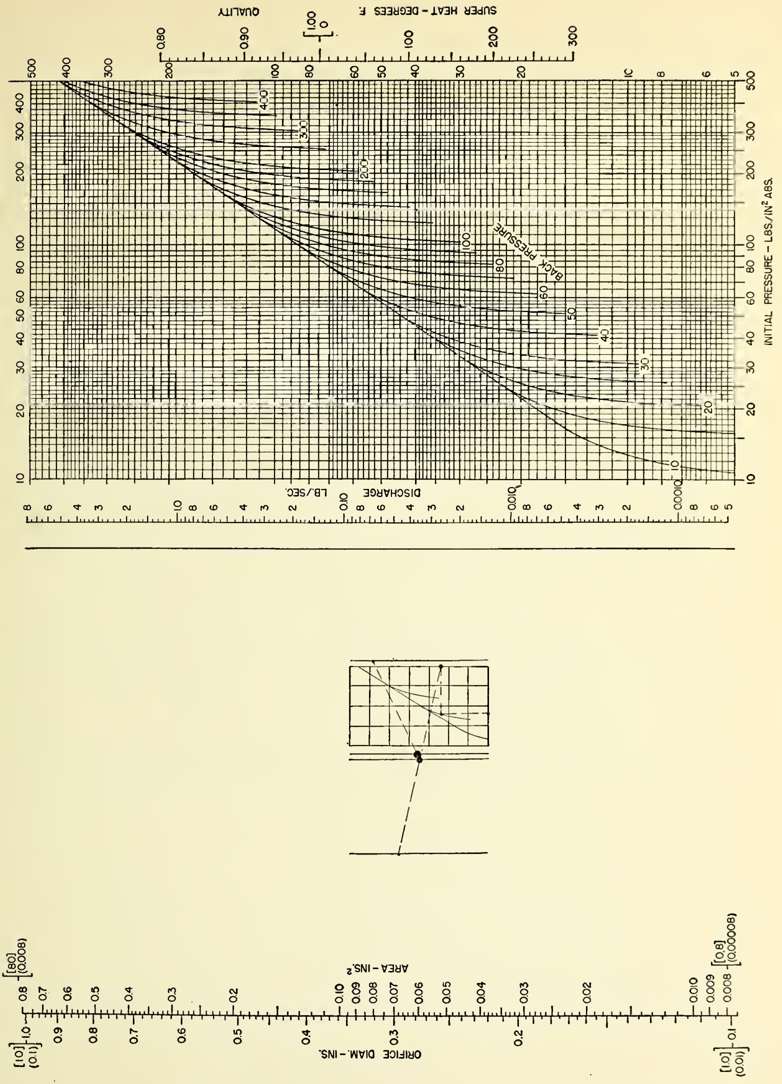
The second step in solution is then to establish a straight line through this point and the scale point corresponding to the known value of (orifice) *Area,* noting the intersection of this line with the blank axis in the center of the chart. A second straight line is then established between this intersection and the scale point corresponding to the known value of *Superheat* or *Quality* on the scale so designated, and the intersection of this line with the *Discharge* scale then gives the solution for the conditions taken as a basis.

By way of illustration, assume an initial pressure of 12 pounds per square inch absolute and a back pressure of 10 pounds absolute, with an orifice having an area of 0.10 square inch, under two steam conditions: (1) 81 per cent quality, (2) having 200° superheat. First, the intersection of

the vertical line for 12 pounds initial pressure with the curve for 10 pounds back pressure is located, and the horizontal line is followed across to the right to the edge of this section to that vertical line.

A straight line through this point and the scale point for 0.10 *Area* is set up, and its intersection with the blank axis is noted. Then a line through this point and the scale point for 0.81 on the *Quality* scale is found to intersect the *Discharge* scale at a point that shows 0.0157 pound per second, the computed value being 0.0154. For the second case, the observed point on the blank axis is connected with the scale point for 200° superheat, and the *Discharge* is read as 0.0123, the computed value being 0.01224.

A key diagram appearing on the chart should serve to guide operations, after a first study of this explanation, in routine use of the chart.


INITIAL PRESSURE - LBS./IN² ABS.
BACK PRESSURE
DISCHARGE LB./SEC.
QUALITY
SUPER HEAT - DEGREES F
AREA - INS.²
ORIFICE DIAM. - INS.

# 66. Steam Pipe Flow

**Method for the determination** of the pressure drop due to friction in the flow of steam in pipes. It is based upon the widely used Unwin formula, wherein the pressure drop in pounds per square inch is

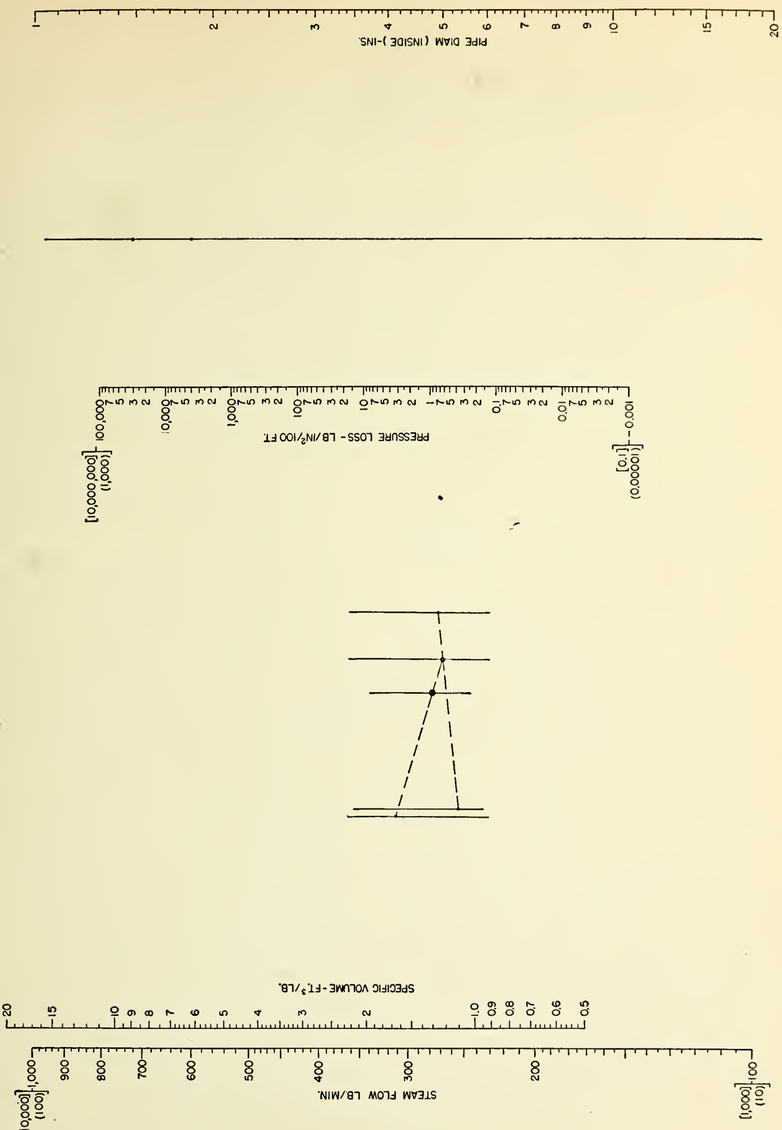
$$P = 0.0001306W^2v\left(\frac{1 + 3.6d}{d^5}\right)L$$

where $W$ is the amount of steam passing through the pipe in pounds per minute, $v$ is the specific volume of the steam in cubic feet per pound, $d$ is the inside diameter of the pipe in inches, and $L$ is the length of the pipe in feet, making allowance for equivalent length for fittings. The chart omits provision for the factor $L$, pressure drops being in pounds per hundred feet of length.

**Procedure.** To use the chart, a straight line is established through the scale points corresponding to the known values on the *Pipe Diameter* and *Specific Volume* scales, respectively, and the intersection of this line with the blank axis is noted. Then a second straight line is fixed by this point and the scale point for the known value on the *Steam Flow, Lb./Min.* scale, and at the intersection of this line with the *Pressure Loss* scale, the solution is read.

To illustrate use of the chart, a determination may be made for the condition of 350 pounds of steam per minute flowing in a line of 5.00-inch inside diameter, the specific volume of the steam being 3.00 cubic feet per pound. First, a line is established through the scale point for 5.00 on the *Pipe Diameter* scale and 3.00 on the *Specific Volume* scale, and the intersection of this line with the blank axis is noted. A second straight line established through this point and the scale point for 350 on the *Steam Flow, Lb./Min.* scale is then found to intersect the *Pressure Loss* scale at the scale point indicating a drop of 2.70 pounds per square inch per 100 feet of pipe length. Straight computation gives the result as 2.6516.

A key diagram on the chart illustrates the sequence of operations and should save frequent reference to this explanation.


STEAM FLOW LB./MIN.
SPECIFIC VOLUME - FT.$^3$/LB.
PRESSURE LOSS - LB/IN$^2$/100 FT.
PIPE DIAM (INSIDE)-INS.

# 67. Throttling Calorimeter

**Method for the determination** of the quality of steam from tests made with the throttling type of calorimeter. This, of course, is the instrument wherein wet steam is throttled from its normal high pressure, at which it is wet, to a lower pressure at which the same amount of heat makes the steam superheated. The calorimeter being insulated, the amount of heat lost in throttling is negligible, so an equation is set up between the heat factors before and after, and this equation is solved for the unknown factor of quality.

The equation of the various heat factors before and after throttling is, of course,

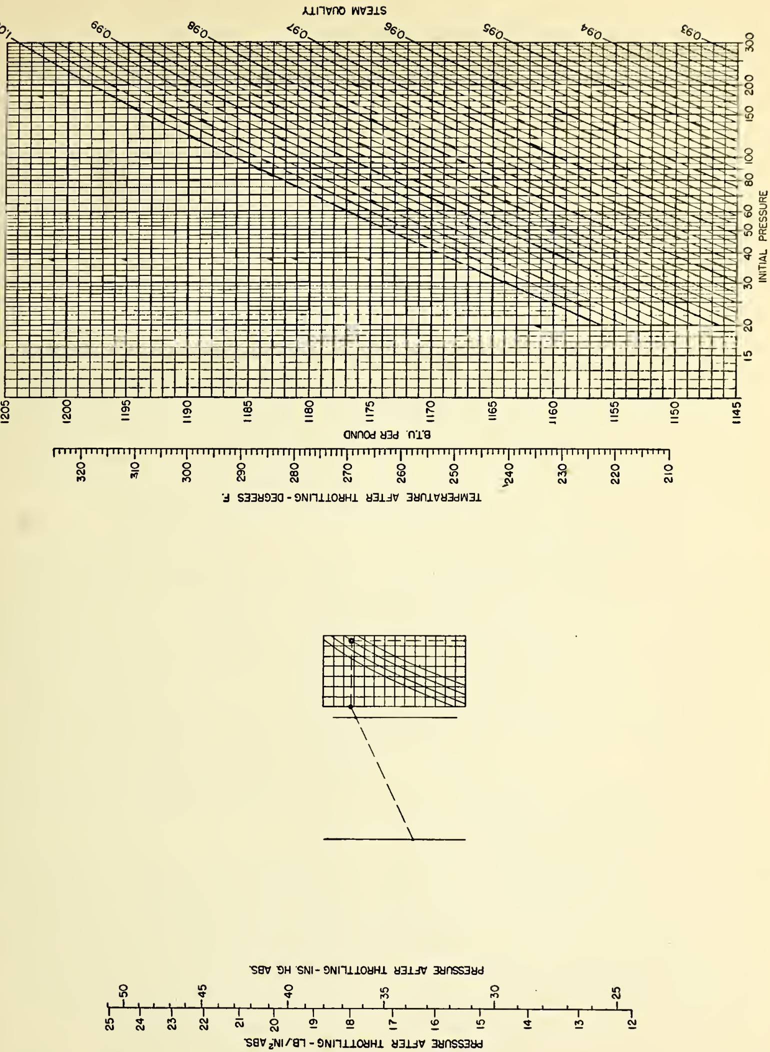
$$X_1L_1 + h_1 = H_2 + c(T_s - T_0)$$

where $X_1$ is the steam quality, $L_1$ is the latent heat in B.T.U. per pound of steam at the initial pressure, $h_1$ is the heat in B.T.U. per pound of water at the saturation temperature corresponding to the initial pressure, $H_2$ is the heat in B.T.U. per pound of dry, saturated steam at the pressure after throttling, $T_0$ is the saturation temperature corresponding to the pressure after throttling, $T_s$ is the actual temperature of the steam after throttling as measured in the calorimeter, and $c$ is the specific heat of steam in the range of temperature between $T_0$ and $T_s$. Under the near-atmospheric conditions that usually obtain, this last factor is generally taken as 0.47.

**Procedure.** The first step in the use of the chart is to establish a straight line through the scale points corresponding to the known values of *Pressure after Throttling* and *Temperature after Throttling* on the scales so designated. The intersection of this line is then noted on the vertical line which forms the left-hand edge of the rectangular-coordinate area at the right side of the main chart. Then at the intersection of the horizontal line (interpolating as necessary) corresponding to the point so found with the vertical line corresponding to the known value of *Initial Pressure* (absolute), the *Steam Quality* is read from the family of curves, again interpolating as necessary.

By way of illustrating use of the chart, assume that test observations show an initial pressure of 200 pounds per square inch absolute and a pressure after throttling of 18 pounds per square inch absolute, the temperature after throttling being 270°F. First, a line is established through the scale points for 18.0 on the *Pressure after Throttling* scale and 270 on the *Temperature after Throttling* scale, and the intersection of this line is noted on the vertical line of the left-hand edge of the rectangular-coordinate system. Then, the horizontal line (interpolating) from this point to the right intersects the vertical line scaled 200 on the *Initial Pressure* scale between the curves for 0.974 and 0.976, and interpolation indicates the *Quality* to be 0.9748, where actual computation gave 0.9747.

A key diagram appears on the chart to serve as a guide in operations, saving reference each time to this text.


STEAM QUALITY
1.00
0.99
0.98
0.97
0.96
0.95
0.94
0.93
INITIAL PRESSURE
15 20 30 40 50 60 80 100 150 200 300
B.T.U. PER POUND
1205 1200 1195 1190 1185 1180 1175 1170 1165 1160 1155 1150 1145
TEMPERATURE AFTER THROTTLING - DEGREES F.
320 310 300 290 280 270 260 250 240 230 220 210
PRESSURE AFTER THROTTLING - INS. HG ABS.
50 45 40 35 30 25
PRESSURE AFTER THROTTLING - LB/IN² ABS.
25 24 23 22 21 20 19 18 17 16 15 14 13 12

# 68. Factor of Evaporation

**Method for the determination** of the factor of evaporation, or the equivalent evaporation from and at 212°F., for operating conditions of boilers within normal range. It also determines simultaneously the total heat absorption per pound of steam in B.T.U.

The factor of evaporation is the ratio existing between the amount of heat added to the water in the boiler under the actual conditions at the beginning and end of the process and the amount of heat required to change the water to steam from and at 212°F. Mathematically, the expression for factor of evaporation is

$$f = (H_2 - H_1)/970.4$$

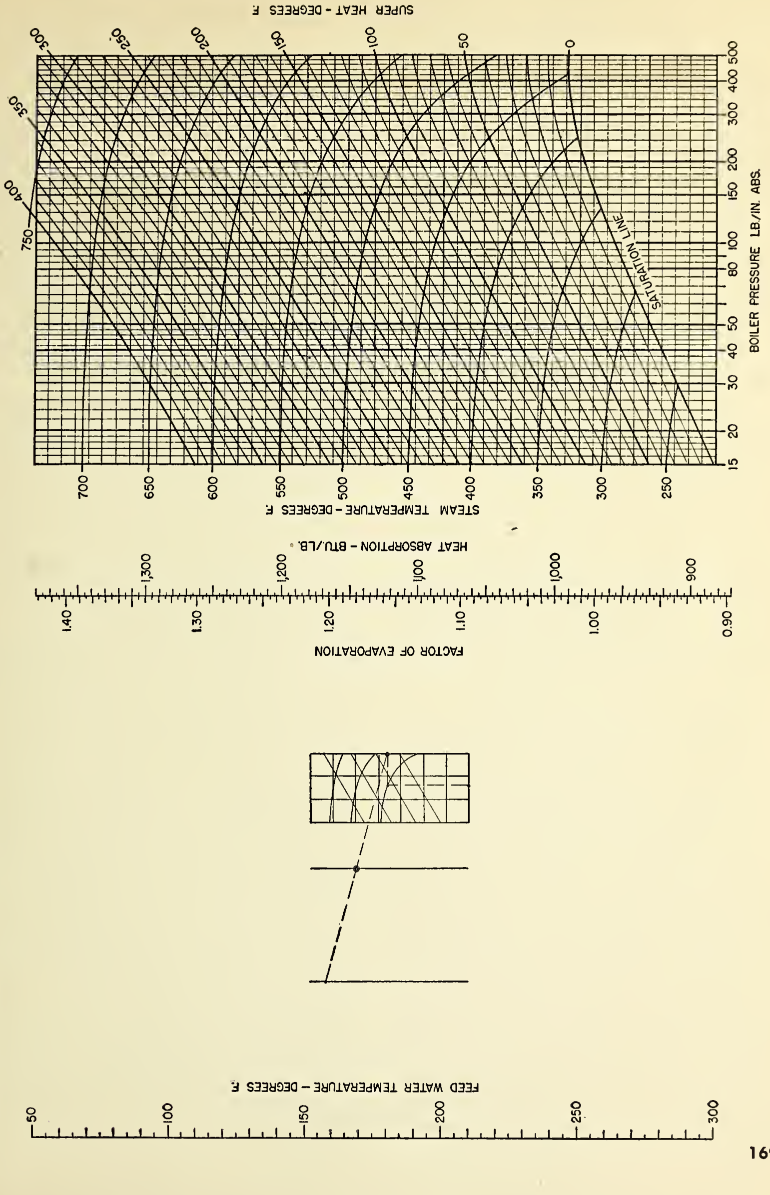
where $H_2$ is the amount of heat per pound of steam under the final conditions of pressure and superheat, and $H_1$ is the amount of heat per pound of water as fed to the boiler, both quantities being in B.T.U. The quantity enclosed in parentheses is the amount of heat added per pound, in the same units.

**Procedure.** The chart consists of two sections, one an intersection-type nomograph and the other an alignment type. The first step in the use of the chart is to locate in the intersection-type (rectangular-coordinate) section at the right a point corresponding to the final steam condition. This is accomplished by locating the intersection of the vertical line representing the scale value of *Boiler Pressure,* with the curve of *Saturation,* the curve of *Steam Temperature,* or the curve of *Superheat,* depending upon conditions and the form in which the known data may be stated.

The second step is to carry across along the horizontal lines (interpolating as necessary) the vertical measure of this point to the vertical line bounding this portion of the chart at its right-hand edge. The third step is then to establish a straight line through this point and the scale point corresponding to the *Feed Water Temperature,* on the scale so designated. Then, at the intersection of this line with the scale in the center of the chart may be read the desired values for *Factor of Evaporation* or *Heat Absorption* per pound.

By way of illustrating the use of the chart, a condition is assumed where the feed-water temperature is 202°F. with final steam at 160 pounds per square inch absolute and a temperature of 400°F. First, the intersection point is located for the vertical line for a *Boiler Pressure* of 160 and the curve of 400 for *Steam Temperature.* Next, the vertical coordinate of this point is followed horizontally across this section to the right-hand edge, where it is noted on the vertical line forming that edge.

Finally, this point and the scale point for 202 on the *Feed Water Temperature* scale are used to establish a straight line. Then, at the intersection of this line with the central scale, the *Factor of Evaporation* is read as 1.0785 and the *Heat Absorption* is read as 1047 B.T.U. per pound. Reference to steam tables, followed by arithmetical computation, shows results of 1.0789 and 1047, respectively.


SUPER HEAT - DEGREES F
0
50
100
150
200
250
300
350
400
750
BOILER PRESSURE LB/IN. ABS.
15
20
30
40
50
80
100
150
200
300
400
500
SATURATION LINE
STEAM TEMPERATURE - DEGREES F
250
300
350
400
450
500
550
600
650
700
HEAT ABSORPTION - BTU./LB.
900
1,000
1,100
1,200
1,300
FACTOR OF EVAPORATION
0.90
1.00
1.10
1.20
1.30
1.40
FEED WATER TEMPERATURE - DEGREES F
50
100
150
200
250
300

# 69. Combustion Air and Carbon Dioxide

**Method for the determination** of the theoretical minimum weight of air required for combustion of a fuel when the chemical analysis is known. The chart also determines the theoretical maximum amount of carbon dioxide which would result if the fuel were completely burned with that amount of air. In addition, it is possible to determine the actual amount of air for lower values for the carbon dioxide content of the flue gases.

One of the two equations upon which the chart is based is that for the theoretical amount of air, where the weight of air in pounds per pound of dry fuel is

$$W_0 = 11.49C + 34.48(H - O/8) + 4.31S$$

where C is the carbon content, H is the hydrogen content, O is the oxygen content, and S is the sulphur content, all as decimal parts of the total dry weight. The carbon dioxide content of the flue gas = $CO_2$ = $2.4C/W$, as theoretical maximum.

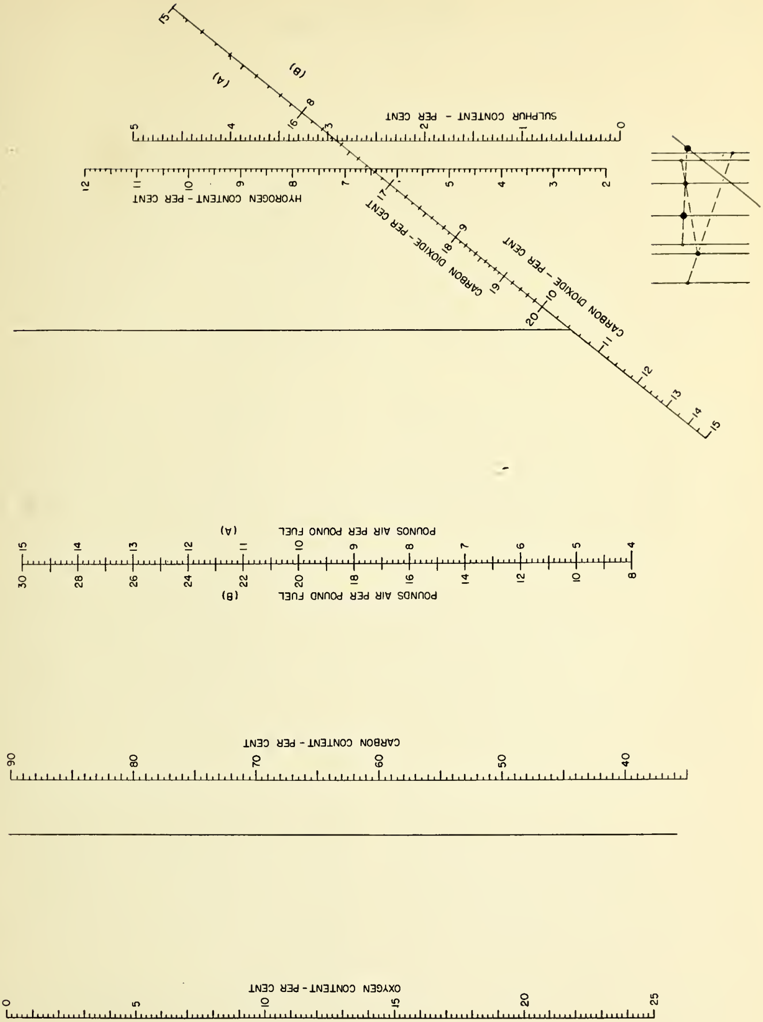
**Procedure.** The use of the chart is in three stages, the first of which is to establish a straight line through the scale points corresponding to the known values for *Sulphur Content* and *Oxygen Content* on the scales so designated. The intersection of this line with the first blank axis, near the left-hand side of the chart, is then noted. A second straight line is then established through this point and the scale point corresponding to the known value of *Hydrogen Content* on *that* scale, and the intersection of this line is noted on the second blank axis, at the right side of the chart.

Then, a third straight line is set through this point and the scale point corresponding to the *Carbon Content,* and on the *A* scalings on the *Pounds Air per Pound Fuel* and *Carbon Dioxide—Per Cent* scales are read the final figures. To find the amount of air for an actual observed amount of carbon dioxide, a straight line through the scale points for the given values for *Carbon Content* and *Carbon Dioxide* will intersect the *Pounds Air per Pound Fuel* scale at a scale value that gives the answer. The *B* scales are added to increase the range of this operation.

To illustrate the use of the chart, conditions are determined for a bituminous coal of 0.777 carbon, 0.049 hydrogen, 0.108 oxygen, and 0.040 sulphur. First, a line is set through 4.0 per cent on the *Sulphur Content* scale and 10.8 on the *Oxygen Content* scale, and its intersection is noted on the blank axis at the left. A second line is then laid through this point and 4.9 on the *Hydrogen Content* scale, and its intersection is noted on the blank axis at the right. A third line through this point and 77.7 on the *Carbon Content* scale is then found to intersect the *Pounds Air per Pound Fuel A* scale at 10.32 and the *Carbon Dioxide—Per Cent* scale at 18.1. The computed values were 10.321 and 18.09, respectively.

If the observed carbon dioxide in the stack gases is 12 per cent, a line is established through the scale point corresponding to this value (with use of the *B* scale necessary), and the scale point for 77.7 on the *Carbon Content* scale, and the amount of air per pound of fuel is read as 15.60, the computed value being 15.54.

To find the weight of the dry flue gases per pound of fuel, the weights of sulphur and carbon per pound of fuel are added, and eight times the weight of hydrogen is deducted.


SULPHUR CONTENT - PER CENT
HYDROGEN CONTENT - PER CENT
CARBON DIOXIDE - PER CENT
CARBON DIOXIDE - PER CENT
(A)
(B)
POUNDS AIR PER POUND FUEL (A)
POUNDS AIR PER POUND FUEL (B)
CARBON CONTENT - PER CENT
OXYGEN CONTENT - PER CENT

# 70. Heat Loss in Flue Gases

**Method for the determination** of the loss due to the heat carried from a boiler in the dry stack gases. It gives this loss in actual heat per pound of fuel and as a percentage of the total heat in the fuel, and it covers a wide range in operating conditions.

The equation upon which the chart is based is the conventional one, wherein the heat loss in B.T.U. per pound of fuel is

$$H = c(T - t)W$$

where $c$ is the specific heat of the stack gases, here taken as 0.24, an average value, $T$ is the flue-gas temperature, $t$ is the temperature of the incoming combustion air, both temperatures being in degrees Fahrenheit, and $W$ is the weight of flue gases per pound of fuel.

It will be noted that the scales for the calorific value of the fuel and for the percentage loss are double-scaled, one set of scalings, $A$, covering the normal range for solid fuels and the second, $B$, covering that for oil fuel. In the use of the chart it is necessary to be consistent in the use of these scales, using $A$ with $A$ and $B$ with $B$.

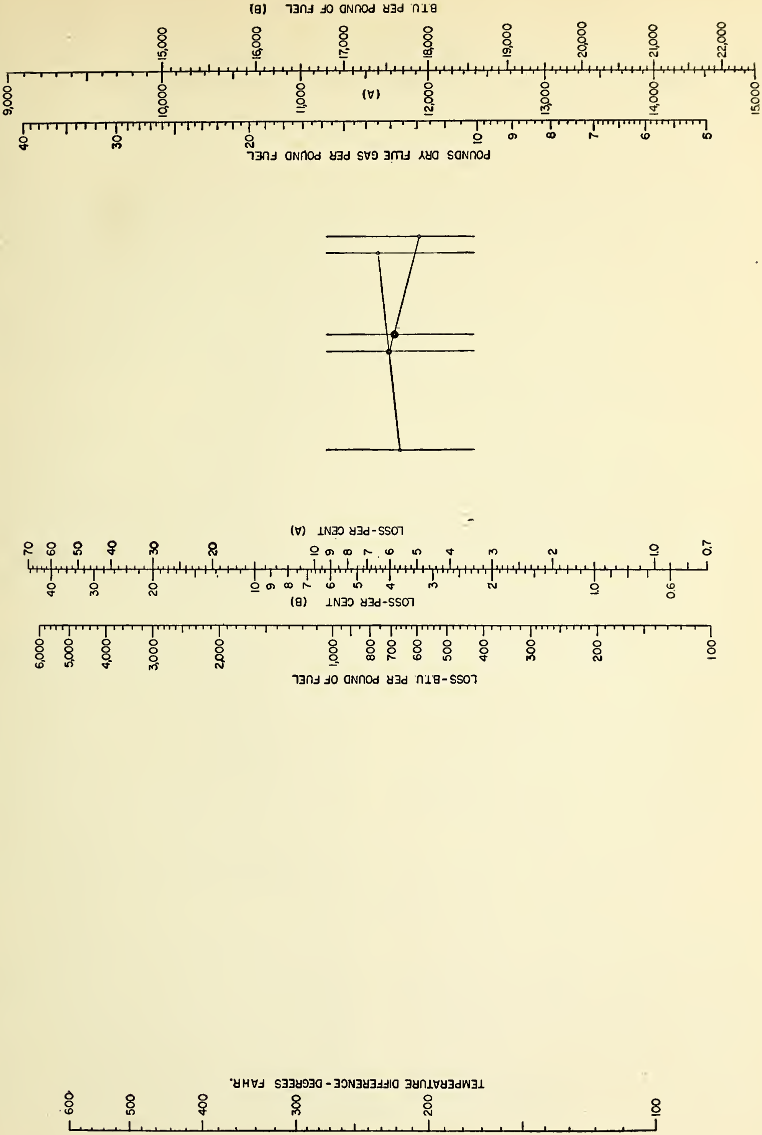
**Procedure.** To use the chart, the first step is to establish a straight line through the scale points corresponding to the known values for *Pounds Dry Flue Gas per Pound Fuel* and *Temperature Difference* on their respective scales as so designated. At the intersection of this line with the scale so designated is read the *Loss—B.T.U. per Pound of Fuel.* Then a second straight line is established through this intersection point and the scale point corresponding to the calorific value of the fuel on the *B.T.U. per Pound of Fuel* scale.

Then, at the scale point at the intersection of this line with the *Loss—Per Cent* scale is read the value of the percentage loss. Again it should be noted that if $A$ scale is used for the calorific value of the fuel, the $A$ scale is used for the percentage loss, and likewise with the $B$ scales.

To illustrate the use of the chart, conditions are assumed where the difference between combustion-air and flue-gas temperatures is 300°F., with 35 pounds of flue gas per pound of fuel having been determined by a stack-gas analysis, the fuel being an oil of a calorific value of 20,000 B.T.U. per pound. Then, the first step is in establishing a straight line through the scale points corresponding to 300 on the *Temperature Difference* scale and 35.0 on the *Pounds Dry Flue Gas per Pound Fuel* scale. At the intersection of this line with the scale so designated the *Loss—B.T.U. per Pound of Fuel* is read as 2510 (computed value 2520).

Then, a second straight line is established through this point and the scale point for 20,000 on the *B.T.U. per Pound of Fuel* scale, which is found on the $B$ scale. Then, the intersection of this line with the *Loss—Per Cent B* scale is at a scale value of 12.6 per cent, this also being the value as computed.

A key diagram on the chart shows the sequence of operations and should save time in avoiding frequent reference to this written discussion.



B.T.U. PER POUND OF FUEL (B)
(A)
POUNDS DRY FLUE GAS PER POUND FUEL
LOSS-PER CENT (A)
LOSS-PER CENT (B)
LOSS-B.T.U. PER POUND OF FUEL
TEMPERATURE DIFFERENCE - DEGREES FAHR.

# 71. Heat Loss by Moisture in Flue Gas

**Method for the determination,** in boiler operation, of the loss attributable to moisture, in B.T.U. per pound of fuel and in per cent. In this loss there are two components, both of which may be evaluated by means of this chart. The first component is the loss due to the moisture, as water, in the fuel as fed to the boiler, and the second is that due to combustion of hydrogen in the fuel, which forms water, which must be heated to the stack-gas temperature.

As to the moisture in the fuel as fired, there can be little question about the heat absorbed in raising it to stack temperature. But as to the product of combustion of the hydrogen, there is a difference of opinion about how this factor should be treated. However, this chart is based upon the more pessimistic view as regards losses, although the difference involved in really quite small.

There are two equations upon which the chart is based, the one for the original moisture in the fuel, and the other for the water created by the combustion of the hydrogen in the fuel. These equations are

$$L_1 = MH_0 \tag{1}$$

and

$$L_2 = 9.01hH_0 \tag{2}$$

where $L_1$ is the heat loss in B.T.U. per pound of fuel due to moisture in the fuel as fed to the boiler, $M$ is the moisture content of the fuel as fed, as a decimal figure, $L_2$ is the loss in B.T.U. per pound of fuel as fed to the boiler, $h$ is the hydrogen content of the fuel, as a fractional quantity, and $H_0$ is the heat required to bring the moisture to the stack conditions.

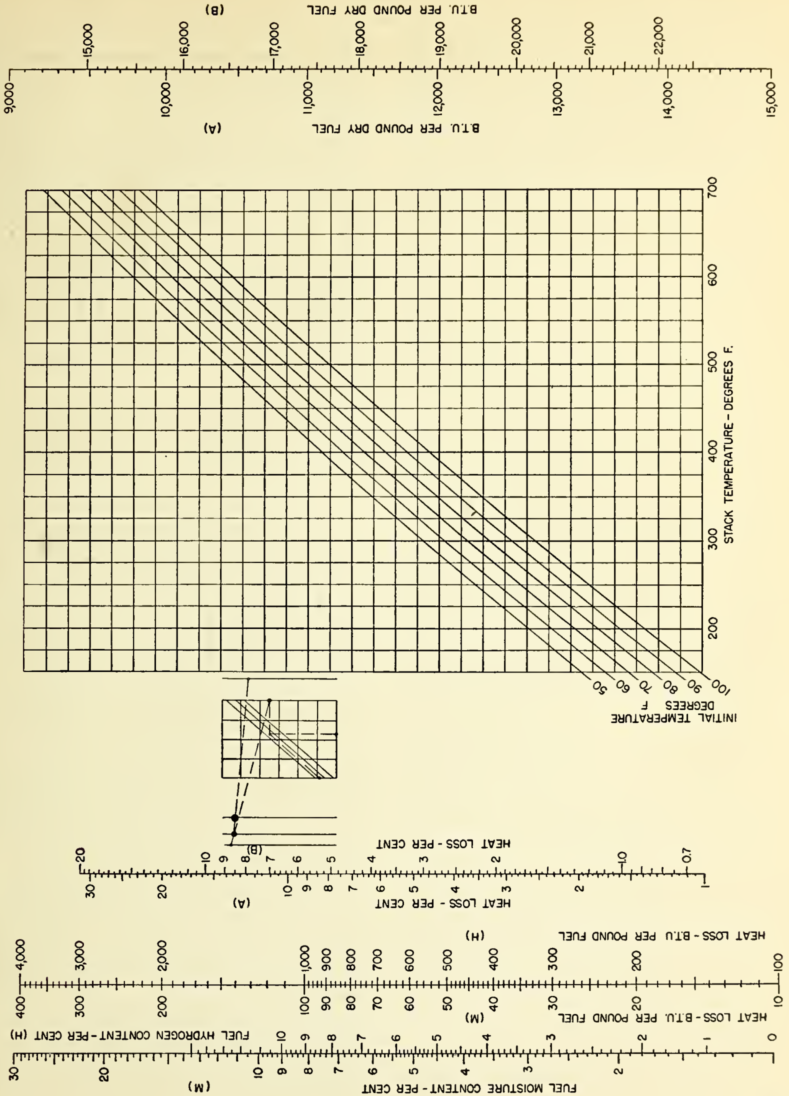
This last factor takes into consideration the heat required to bring the moisture to boiling point, to evaporate it, and then to superheat it to stack temperature. The amount of heat required for this purpose per pound of moisture in the fuel is evaluated in B.T.U. as

$$H_0 = 970.4 + 180 - h_0 + 0.47(t_s - 212)$$

where $h_0$ is the heat in the water in B.T.U. per pound at the temperature as fired and $t_s$ is the temperature of the stack gases in degrees Fahrenheit. The factor 0.47 is taken as the average specific heat in the range covered.

**Procedure.** To use the chart, for either of the two procedures for which it is designed, it is first necessary to arrive at an evaluation of the $H_0$ term. Thus, the first step is to locate in the intersection-type-nomograph section (of rectangular coordinates) the intersection of the vertical line corresponding to the known value of *Stack Temperature* and the line (slightly curved) corresponding to the known value of *Initial Temperature,* interpolating as necessary in each case.

Then, from this point of intersection, the vertical height of this point is carried across this portion of the chart to the right-hand edge of this coordinate section, and this intercept is there noted. This point is then used for both of the procedures for which the chart is designed. To simplify later discussion, this point is referred to as $X$.


B.T.U. PER POUND DRY FUEL (B)
B.T.U. PER POUND DRY FUEL (A)
STACK TEMPERATURE - DEGREES F.
INITIAL TEMPERATURE DEGREES F
HEAT LOSS - PER CENT (B)
HEAT LOSS - PER CENT (A)
HEAT LOSS - B.T.U. PER POUND FUEL (H)
HEAT LOSS - B.T.U. PER POUND FUEL (M)
FUEL HYDROGEN CONTENT - PER CENT (H)
FUEL MOISTURE CONTENT - PER CENT (M)

Now, to determine the loss due to moisture in the fuel as fired, a straight line is established between this point *X* and the scale value corresponding to the known value of *Moisture Content,* and at the intersection of this line with the *Heat Loss—B.T.U. per Pound Fuel* (here using the *M* scale) the loss is read, and that point is noted. Then, by establishing a second straight line through this point and the scale point corresponding to the calorific value of the fuel, in *B.T.U. per Pound Dry Fuel,* an intersection may be noted with the *Heat Loss—Per Cent* scale. It should be borne in mind, however, that, if the calorific value of the fuel is taken on the *A* scale, the per cent reading should also be on the *A* scale, and likewise for *B*. Further, in determinations for loss by moisture in the fuel as fired, *the percentage figures of the chart must be divided by 10.*

For the second use of the chart, the determination of losses due to moisture created by combustion of hydrogen to form water, it is necessary to refer back to the point established as *X*. With this point and the scale point corresponding to the *Fuel Hydrogen Content,* on the scale so designated, a straight line is established, and its intersection is noted at the *Heat Loss—B.T.U. per Pound Fuel* axis, this time using the *H* scaling. Then, another straight line through this point and the scale point corresponding to the known calorific value of the fuel on the *B.T.U. per Pound Dry Fuel* scale will give, at the intersection with the *Heat Loss—Per Cent* scale, the value of that quantity. Again, if the *A* scaling is used for one of the figures, the *A* scaling must also be used for the other, and likewise for *B*.

By way of illustrating the use of the chart, a set of conditions is assumed wherein the temperature of the incoming fuel is 60°F. and the stack temperature is 600°F. It is also taken that the moisture content of the fuel as fired is 18.0 per cent and the hydrogen content of the fuel (dry) is 10.0 per cent.

The first step for either determination is to note the intersection, in the rectangular-coordinate section of the chart, of the vertical line corresponding to 600 on the *Stack Temperature* scale and the curve for 60° *Initial Temperature.* This intercept is then carried horizontally across this section of the chart to the right-hand edge, and there a point is noted, as explained above, as *X*.

Assuming the next step is to find the losses due to the moisture in the fuel as fired, a straight line is then established through point *X* and the scale point for 18.0 on the *Fuel Moisture Content* scale. Then the intersection of this line is noted on the *Heat Loss* scale *M* as 234 B.T.U. (computed value being 234.846). Another straight line is then established through this point and the scale point corresponding to the calorific value of the fuel in B.T.U. per pound, assumed as 12,000, and the intersection of this line with the *Heat Loss—Per Cent* scale is read at 19.5 per cent. But, as mentioned before, the percentage values for fuel-moisture loss must be taken as the scale value divided by 10.0; hence the result is 1.95, where actual computation gives a value of 1.950.

Now, turning to loss by hydrogen in the fuel, the start is again at point *X*. Through this point and the scale point corresponding to the *Fuel Hydrogen Content* another straight line is established, which is found to intersect the *Heat Loss* scale at a point indicating a loss of 1175 B.T.U. per pound of fuel, on the *H* scaling, actual computation having given the figure of 1175.27.

Then, another straight line is fixed through this point and the scale point corresponding to the calorific value of the fuel, still assumed as 12,000 B.T.U. per pound, and the intersection of this line with the *Heat Loss—Per Cent* scale gives a value of 9.83 per cent, where computation results in 9.79 per cent.

A key diagram on the chart illustrates the procedure to be followed in use of the chart, which should obviate frequent recourse to this text. But it should be borne in mind that always the percentage determinations from the chart with regard to moisture in the fuel as fired must be multiplied by 0.10.

# 72. Heat Loss by Incomplete Combustion

**Method for the determination** of the heat loss occasioned by part of the carbon in fuel being burned only to carbon monoxide instead of being completely burned to carbon dioxide. The equation upon which the chart is based is that where the heat loss in B.T.U. per pound of dry fuel is

$$L = 10{,}160(C)(CO)/(CO + CO_2)$$

where C is the carbon content of the dry fuel, as the fractional part by weight, CO is the measure of the carbon monoxide in the flue gases, as a fractional part by volume, and $CO_2$ is the measure of the carbon dioxide in the flue gases, also as a fractional part by volume.

**Procedure.** The first step in the use of the chart is to locate, in the intersection-type-nomograph section (rectangular coordinates) at the left side of the chart, the point of intersection of the vertical line whose scale value corresponds to the observed value of *Carbon Dioxide in Flue Gas* with the curve whose scale value corresponds to the observed value of *Carbon Monoxide in Flue Gas,* interpolating as necessary. From this point, the vertical distance is carried horizontally along the horizontal lines to the right-hand edge of this portion of the chart, to the line which forms the right-hand edge.

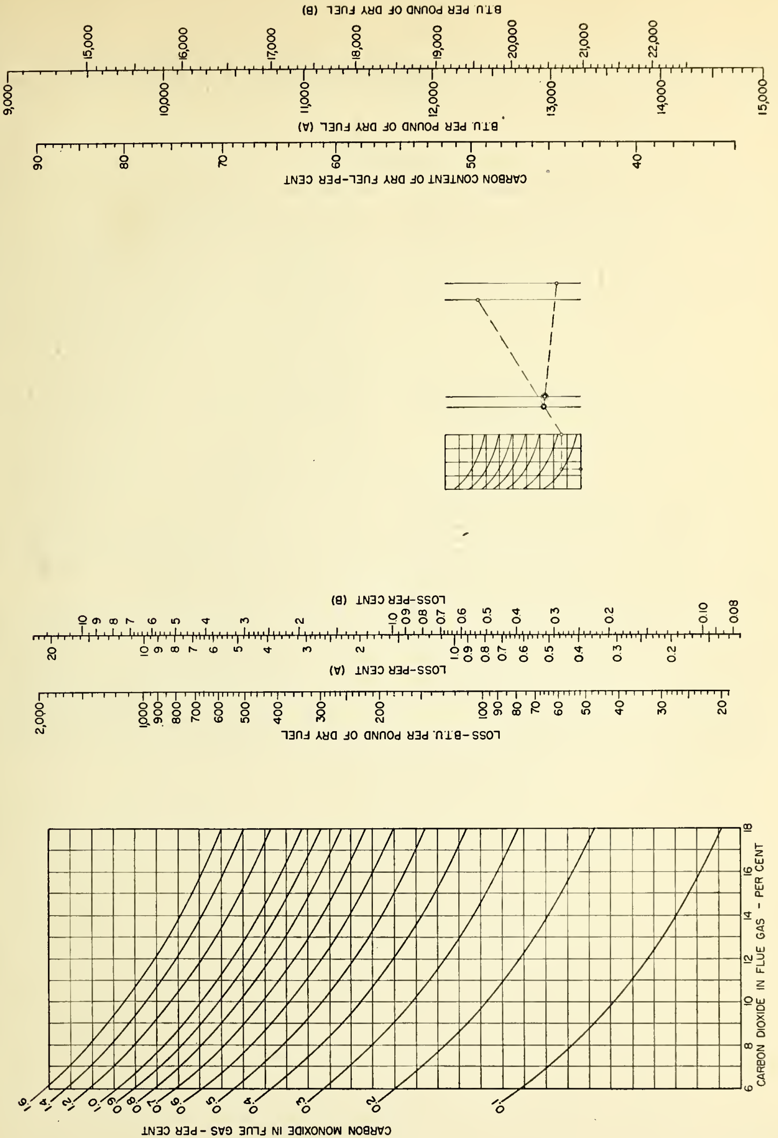
Then, a straight line is established through this point and the scale point corresponding to the known value of *Carbon Content of Dry Fuel,* on the scale so designated. Then, at the intersection of this line with the *Loss—B.T.U. per Pound of Dry Fuel* scale is read that quantity.

To determine percentage loss, a second straight line is fixed by this point and the scale point corresponding to the known figure for *B.T.U. per Pound of Dry Fuel,* on the scale so designated, and at the intersection of this line with the *Loss—Per Cent* scale is read the desired figure. It should be noted that these last two scales are double-scaled, with *A* and *B* scales, which should be used with consistency. Thus, if the fuel calorific value is taken on the *A* scale, the percentage loss must also be read on its *A* scale, and similarly for *B*.

To illustrate use of the chart, a condition is assumed where a fuel of 0.80 carbon content, with a calorific value of 12,000 B.T.U. per pound is burned to conditions where the stack gases show 0.3 per cent carbon monoxide and 14.0 per cent carbon dioxide. First, the intersection is noted for the vertical line of 14.0 for *Carbon Dioxide in Flue Gas* and the curve for 0.3 per cent *Carbon Monoxide in Flue Gas.* Then follow horizontally to the right side of this section of the chart and note the intercept on the vertical line that forms that edge.

Next, a straight line is established through this point and the scale value for 80.0 on the *Carbon Content of Dry Fuel—Per Cent* scale, and at the intersection of this line with the *Loss—B.T.U. per Pound of Dry Fuel* scale is read 171, computed as 170.53. A second straight line fixed by this point and the scale point for 12,000 on the *B.T.U. per Pound of Dry Fuel* scale *A* is then found to intersect the *Loss—Per Cent* scale *A* at a scaling of 1.43 per cent, the computed value being 1.4211 per cent.

The key diagram should serve to assist in use of the chart, in showing sequence of operations at a glance.


B.T.U. PER POUND OF DRY FUEL (B)
B.T.U. PER POUND OF DRY FUEL (A)
CARBON CONTENT OF DRY FUEL-PER CENT
LOSS-PER CENT (B)
LOSS-PER CENT (A)
LOSS-B.T.U. PER POUND OF DRY FUEL
CARBON DIOXIDE IN FLUE GAS - PER CENT
CARBON MONOXIDE IN FLUE GAS - PER CENT

# 73. Heat Loss by Combustible in Refuse

**Method for the determination** of the heat loss represented by unburned fuel in furnace refuse. The chart is based upon the usual equation for such determinations, wherein the heat loss in B.T.U. per pound of dry fuel is

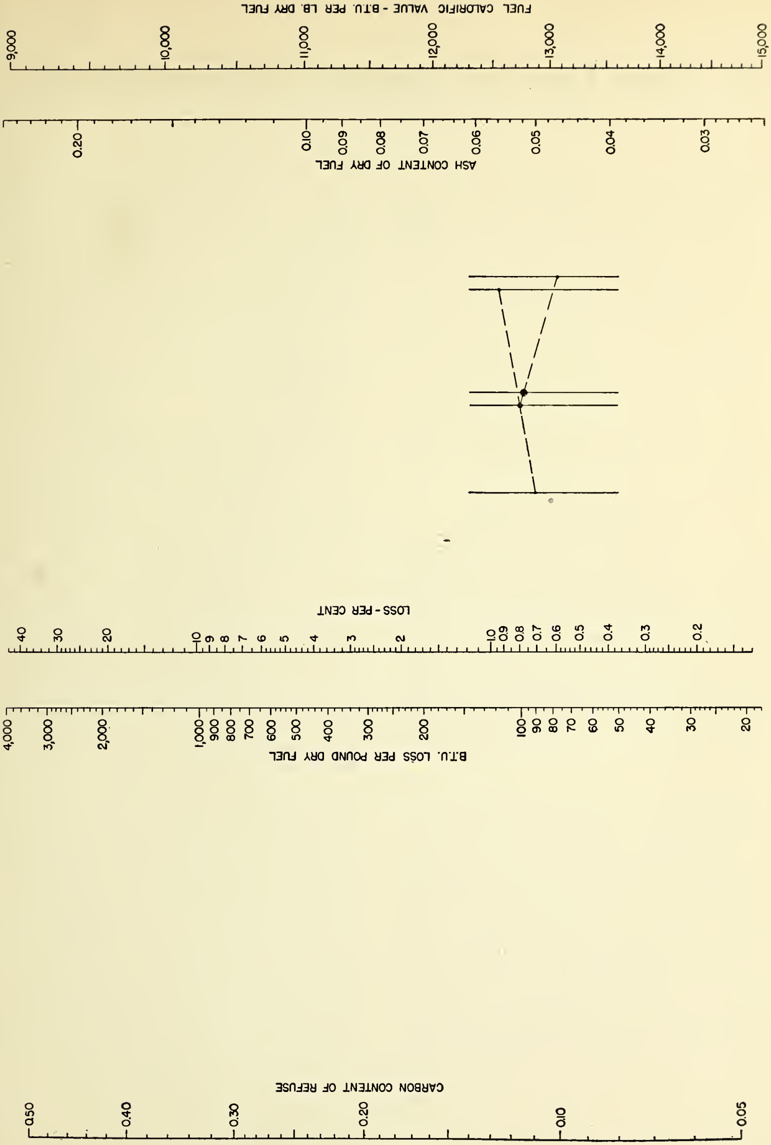
$$L = 14{,}600AC/(1 - C)$$

where $A$ is the ash content per pound of dry fuel and C is the carbon content per pound of furnace refuse.

**Procedure.** Use of the chart is in two stages. First, a straight line is established through the scale points corresponding to known values of *Ash Content of Dry Fuel* and *Carbon Content of Refuse,* on the scales so designated, and the intersection of this line with the *B.T.U. Loss per Pound Dry Fuel* scale gives the measure of that quantity. Then, a second straight line is fixed by this point and the scale point corresponding to *Fuel Calorific Value,* on the scale so marked, and at the intersection of this line with the *Loss—Per Cent* scale is read the percentage loss.

To illustrate the use of the chart, a set of conditions is assumed where analysis shows 8.0 per cent carbon in the refuse, and analysis of the dry fuel showed 8.0 per cent of ash, the calorific value of the fuel being 14,000 B.T.U. per pound. First, a straight line is established through the scale points for 0.08 on the *Carbon Content of Refuse* scale and for 0.08 on the *Ash Content of Dry Fuel* scale. This line is found to intersect the *B.T.U. Loss per Pound Dry Fuel* scale at a scale point corresponding to 102 B.T.U., the computed figure having been found to be 101.616.

Next, a second straight line is fixed by this point and the scale point for 14,000 on the *Fuel Calorific Value* scale. This line is found to intersect the *Loss—Per Cent* scale at a scale point corresponding to 0.725 per cent, where actual computation gave 0.7258 per cent.


FUEL CALORIFIC VALUE - B.T.U. PER LB. DRY FUEL
ASH CONTENT OF DRY FUEL
LOSS - PER CENT
B.T.U. LOSS PER POUND DRY FUEL
CARBON CONTENT OF REFUSE

# 74. Heat Loss by Sensible Heat in Refuse

**Method for the determination** of heat loss in combustion by reason of the sensible heat in the refuse, which has had to absorb otherwise useful heat in order to come to the temperature at which it is removed from the furnace. The chart evaluates this loss both in actual heat units and as a percentage of the total heat in the fuel.

The basic equation for the chart is the usual one for finding this quantity, the heat loss in B.T.U. per pound of dry fuel fired being

$$L = TA\left(0.529 + \frac{0.572\ \mathrm{C}}{1 - \mathrm{C}}\right)$$

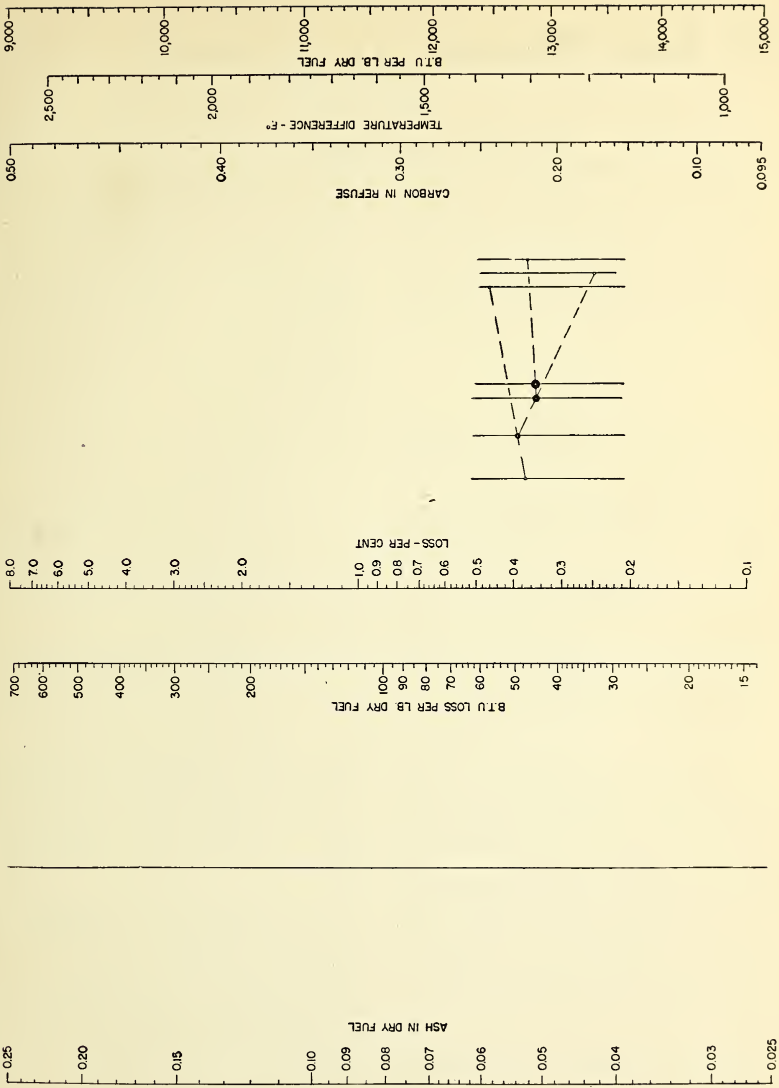
where $T$ is the difference in degrees Fahrenheit between the temperature of the fuel as fired and the temperature of the refuse as removed from the furnace, $A$ is the ash content of the dry fuel, expressed as a decimal, and C is the carbon content of the refuse. 0.529 and 0.572 have been taken as the specific heats of ash and carbon, respectively.

**Procedure.** The first step in the use of the chart is to establish a straight line through the scale points corresponding to the known values of *Ash in Dry Fuel* and *Carbon in Refuse,* on the scales so designated. Then, at the intersection of this line with the blank axis, the intercept is noted. A second straight line is fixed by this point and the appropriate scale point on the *Temperature Difference* scale, and then at the intersection of this line with the *B.T.U. Loss per Pound Dry Fuel* scale is read the value of this loss. Then, a third straight line is fixed by this point and the *B.T.U. per Pound Dry Fuel* scale, and at the intersection of this line with the *Loss—Per Cent* scale the loss is read in per cent.

To illustrate the use of the chart, a set of conditions is assumed where, with a fuel having a calorific value of 14,000 B.T.U. per pound, dry, the ash content of the dry fuel has been determined as 8.0 per cent, and the amount of carbon in the refuse as 30.0 per cent, while the difference between the temperature of the fuel as fired and the temperature of the refuse as removed from the furnace is 1200°F.

First, a straight line is established through the scale point for 0.30 on the *Carbon in Refuse* scale and 0.08 on the *Ash in Dry Fuel* scale. The intercept of this line is noted on the blank axis and a second straight line is established by this point and the scale point for 1200 on the *Temperature Difference* scale. This line is found to intersect the *B.T.U. Loss per Pound Dry Fuel* scale at a scale point corresponding to 74.3 B.T.U., whereas the computed figure is 74.286. Then, a third line, fixed by this point and the scale point for 14,000 on the *B.T.U. per Pound Dry Fuel* scale, is found to intersect the *Loss—Per Cent* scale at a scale point of 0.53 per cent, the computed value being 0.5307 per cent.

A key diagram on the chart illustrates the sequence of operations necessary and should obviate frequent reference to this text.


B.T.U PER LB. DRY FUEL
9,000 10,000 11,000 12,000 13,000 14,000 15,000
TEMPERATURE DIFFERENCE - F°
2,500 2,000 1,500 1,000
CARBON IN REFUSE
0.50 0.40 0.30 0.20 0.10 0.095
LOSS - PER CENT
8.0 7.0 6.0 5.0 4.0 3.0 2.0 1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1
B.T.U LOSS PER LB. DRY FUEL
700 600 500 400 300 200 100 90 80 70 60 50 40 30 20 15
ASH IN DRY FUEL
0.25 0.20 0.15 0.10 0.09 0.08 0.07 0.06 0.05 0.04 0.03 0.025

# 75. Boiler Efficiency

**Method for the determination** of the heat-absorption efficiency of boilers, which, in combination with the foregoing charts, rounds out a series which covers the main items of a boiler heat balance. Two equations form the basis for this chart, the first of which is for heat absorption by the boiler per pound of fuel.

$$H_2 = 970.4FW$$

where $F$ is the factor of evaporation and $W$ is the actual evaporation in pounds of water per pound of fuel.

The second equation is the actual efficiency equation, where the efficiency, in per cent, is

$$E = 100H_2/H_1$$

where $H_2$ is the heat absorption per pound of fuel and $H_1$ is the calorific value of the fuel itself. $H_2$ is based upon the first stage mentioned above, the heat values being in B.T.U.

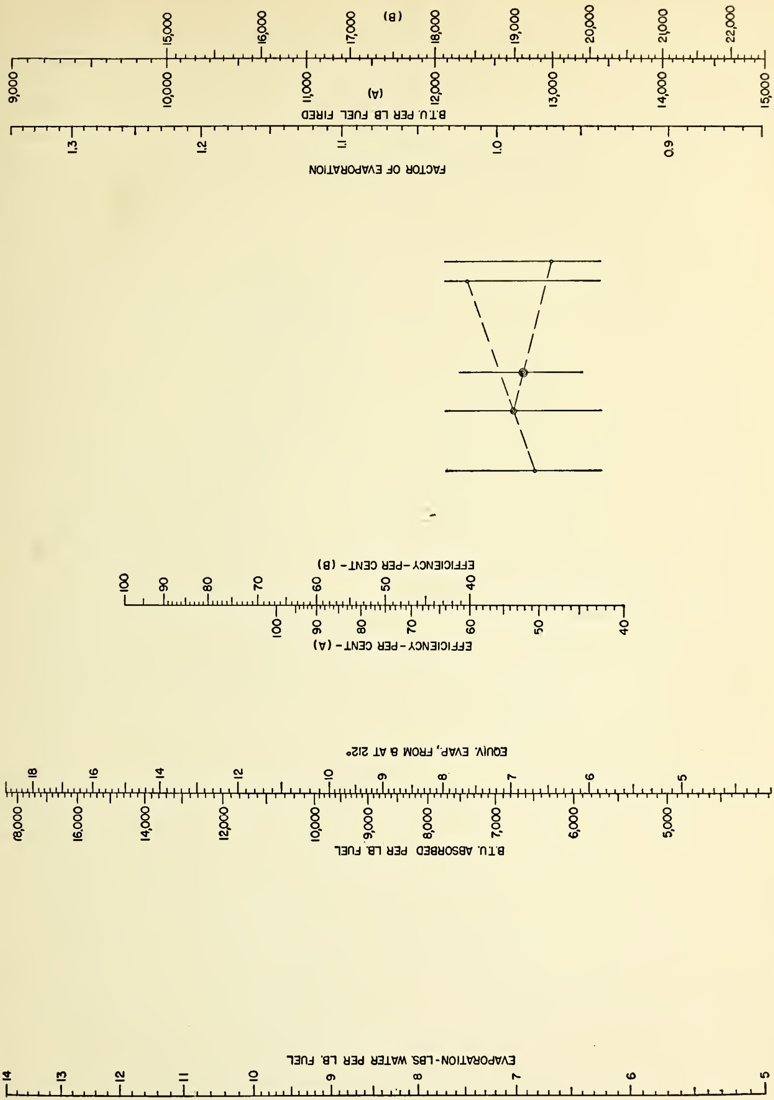
**Procedure.** To use the chart, the first stage consists in establishing a straight line through the scale points corresponding to the known values of *Evaporation—Pounds Water per Pound Fuel* and *Factor of Evaporation.* Then, at the intersection of this line with the double-scaled *B.T.U. Absorbed per Pound Fuel* and *Equivalent Evaporation* scale, either of those items may be read on the scale points of the intersection.

Next, a second straight line is established through this point and the scale point corresponding to the known value for *B.T.U. per Pound Fuel Fired,* and at the intersection of this line with the *Efficiency* scale may be read the figure for efficiency of heat absorption. Note must be taken that the scale for fuel calorific value is double-scaled, each scale covering a different section of the range of this variable, one section being designated as *A* and the other as *B.* Then, if the calorific value is taken on the *A* scale, the efficiency must be read on the *A Efficiency* scale, and likewise with *B.*

To illustrate the use of the chart, a set of conditions is assumed, where the factor of evaporation is 1.10, the actual evaporation is 9.00 pounds of water per pound of fuel, and the calorific value of the fuel is 14,000 B.T.U. per pound.

First, a straight line is established through the scale points for 1.10 on the *Factor of Evaporation* scale and 9.00 on the *Evaporation—Pounds Water per Pound Fuel* scale. Then, on the double-scaled axis, the intersection of this line is at scale points corresponding to 9.90 pounds per pound as the *Equivalent Evaporation* (the true figure) and 9610 *B.T.U. Absorbed per Pound Fuel,* the computed value being 9607.

A second straight line is then fixed by this point and the scale point for 14,000 on the *B.T.U. per Pound Fuel* scale *A,* and the intersection of this line with the *Efficiency* scale *A* is at a scale point indicating 68.7 per cent, the computed value being 68.64.


B.T.U. PER LB FUEL FIRED
(A)
(B)
FACTOR OF EVAPORATION
EFFICIENCY–PER CENT–(B)
EFFICIENCY–PER CENT–(A)
EQUIV. EVAP., FROM & AT 212°
B.T.U. ABSORBED PER LB. FUEL
EVAPORATION-LBS. WATER PER LB. FUEL

# Group VI
# ELECTRICAL CHARTS

This group comprises 17 charts having to do with the solution of problems in the electrical field. The first five of the group are of rather general interest to engineers in various fields, treating of voltage drop in conductors, power measurements, power-factor correction, etc. These are problems which, although they are electrical, are frequently met in engineering fields other than *purely* electrical.

The remaining 12 charts of this group, however, are in a rather specialized field of electrical engineering. They have to do with solution of problems in electrical transmission lines, and while the field of their use may be narrow, their value in saving time makes it seem worth while to include them in this volume. They are based upon the basically exact equations rather than any of the various approximate solutions, but those exact equations involve so much tedious computation that they are ordinarily used only when the results are of primary importance.

These charts, however, reduce the time of solution by the exact basis to minutes where it would be hours for actual computation. Many of the quantities involved are mathematically imaginary or complex, but the charts have been designed to handle such quantities, and their transformation from one system of measure to another, with great ease.

## Charts in Group VI

# 76. Voltage Drop in Conductors

**Method for the determination** of the line drop due to ohmic resistance, disregarding inductance and capacitance effects. It thus applies to comparatively short lines, such as distribution branch circuits, where inductance and capacitance effects are negligible. Under other conditions, the solution of such problems is far more complicated, and recourse should be had to Charts 81 to 92, which *do* take into account all conditions.

As to this chart, however, a great percentage of the problems to which it applies have to do with direct current or single-phase alternating current, where the circuit consists of but two conductors. Accordingly, the chart takes into consideration the distance from supply to load, rather than the total length of conductor. Where the drop in each conductor must be found, it will be one-half the result obtained from this chart.

The chart is based upon the conventional figures stemming from the usual assumption that the resistance of a copper conductor is 10.37 ohms per mil-foot. For a two-conductor circuit, then, the resistance would be 20.73 ohms per mil-foot (of distance to the load), and the voltage drop would be this quantity multiplied by the value of current (voltage drop being equal to product of current and resistance). The equation for voltage drop for a two-wire circuit would then be that line drop in volts is

$$E = 20.73LI/A$$

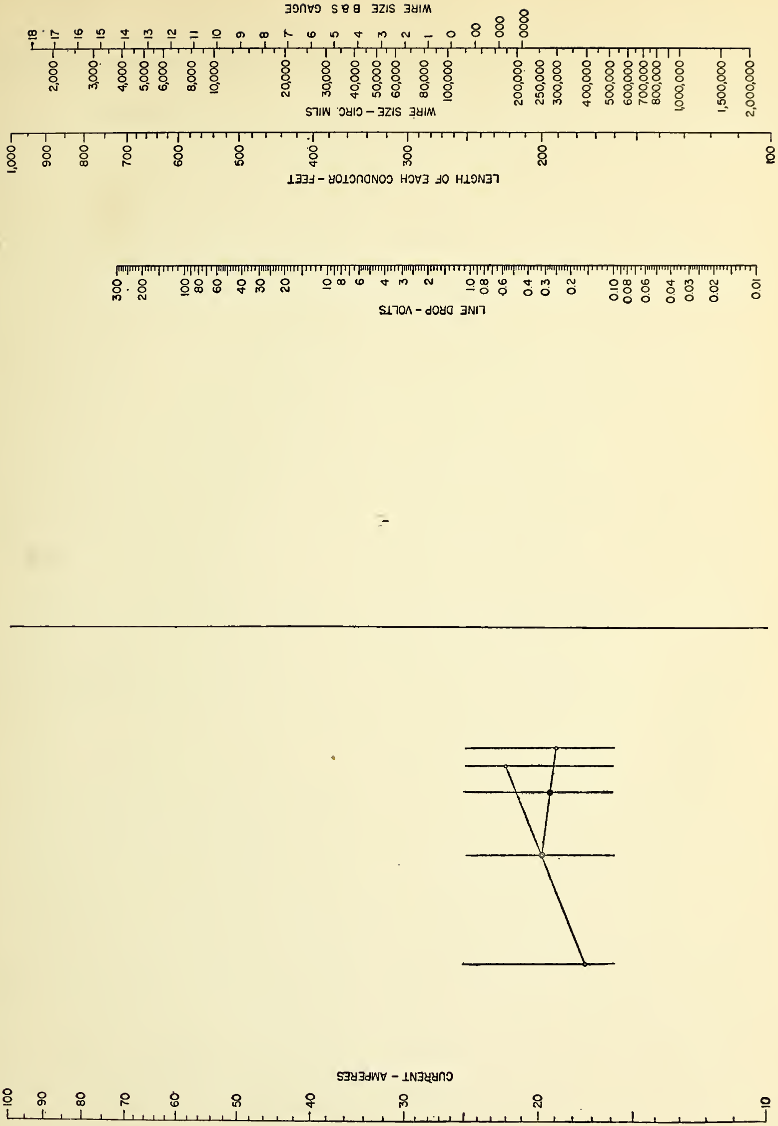
where $L$ is the length of two-conductor circuit in feet, $I$ is the line current in amperes, and $A$ is the cross-sectional area of the conductor in circular mils.

**Procedure.** To use the chart to determine line drop, the first step is to establish a straight line through the scale points corresponding to the known values of *Current—Amperes* and *Length of Each Conductor* on the scales so designated. The intersection of this line with the blank axis is then noted, and a second straight line is then fixed by this point and the scale point corresponding to the *Wire Size,* on the scale so marked and scaled with regard to area as well as gauge number. Then, *Line Drop—Volts* is read at the intersection of this line with the scale so marked.

If the line drop is limited and the wire size is to be determined, the second stage in the use of the chart is to fix the second line by the point on the blank axis and the scale point corresponding to the permissible *Line Drop* value, and at the intersection of this line with the *Wire Size* scale the result is read.

To illustrate the use of the chart, a condition is assumed where the line current is 80.0 amperes, the length of (two-conductor circuit) line is 400 feet, and the wire size is No. 2 A.W.G. First, a straight line is established through the scale point for 80.0 on the *Current* scale and 400 on the *Length of Each Conductor* scale, and the intersection of this line with the blank axis is noted.

Then, a second line is fixed through this


WIRE SIZE B&S GAUGE
WIRE SIZE — CIRC. MILS
LENGTH OF EACH CONDUCTOR – FEET
LINE DROP – VOLTS
CURRENT – AMPERES

point and the scale point for No. 2 on the *Wire Size* scale, and the intersection is noted on the *Line Drop* scale as 10.2 volts. By actual computation, the result is 10.195.

A key diagram on the chart illustrates procedure in use of the chart and should serve to save frequent reference to this text.

# 77. Power Measurement by Watt-hour Meter

**Method for the determination** of the electrical power in kilowatts or equivalent electrical horsepower input to a circuit from observations of a watt-hour meter. Such instruments usually bear, either on the meter disc or on the name plate, a figure indicating the number of watt-hours per revolution of the disc, and this is used as the basis for evaluating the power by measuring the time required for the disc to make a number of revolutions and from this arriving at the average rate of the disc in revolutions per minute through the period of observation.

Derivation of the equation for the power so indicated is a simple matter, and it is necessary to go no further than to give the equation, and upon it the chart is based. The average power through the meter for the period of observation, in kilowatts, is

$$Kw = 0.06C_wC_tN$$

where $C_w$ is the disc constant of the watt-hour meter, in watt-hours per revolution, $C_t$ is the product of the ratios of current and potential transformers, if any, and $N$ is the speed of the disc rotation in revolutions per minute. Naturally, if there are no instrument transformers, $C_t$ will be 1.00, and if either current or potential transformer is absent, $C_t$ will be the ratio of the other.

**Procedure.** To use the chart, a straight line is first established through the scale points corresponding to the known values for *Meter Disc Constant* and *Product of Instrument Transformer Ratios,* on the scales so designated. At the intersection of this line with the blank axis is noted a point which is a measure of the disc constant for the installation, and the same point is used for any studies having to do with the plant under various conditions of loading.

The second step is to establish a second straight line through this point and the scale point corresponding to the observed disc-rotation speed on the *Disc R.P.M.* scale. Then, at the intersection of this line with the central scale the average power may be read on either the *Average Kilowatts Load* or the *Average Horsepower—(Electrical)* scale. If the load is a motor, the shaft horsepower may then be determined from its efficiency characteristic.

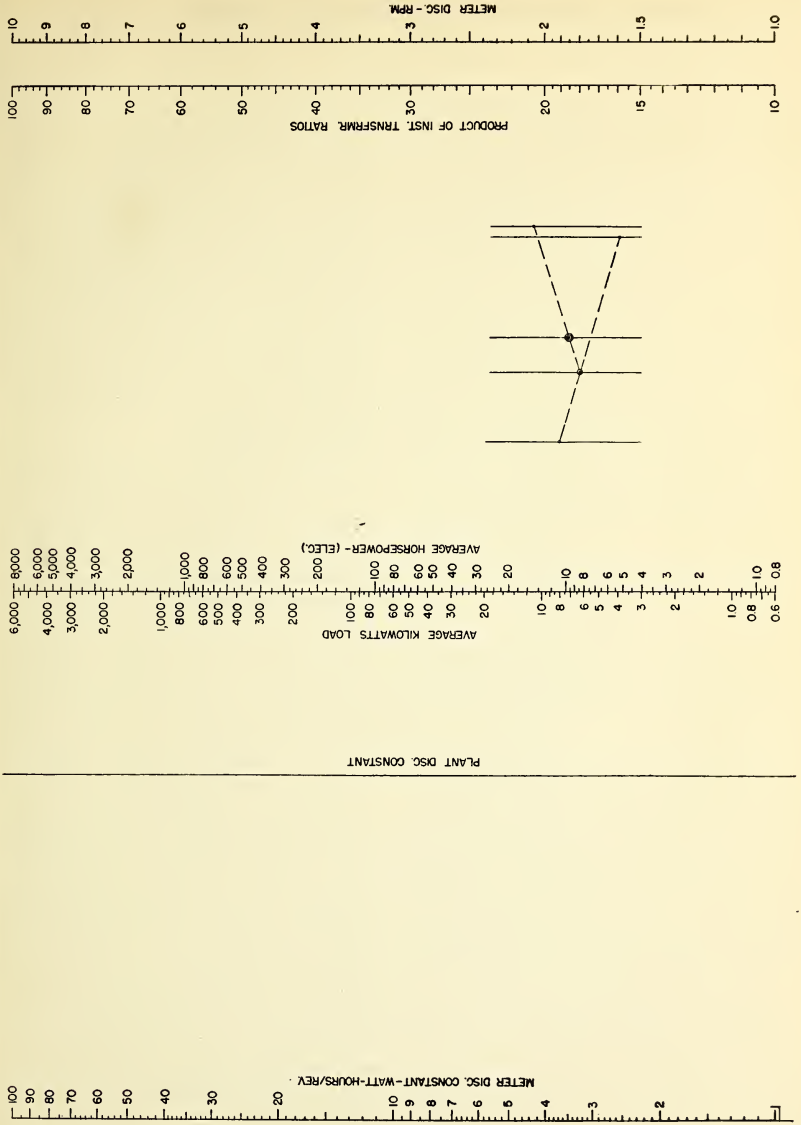
To illustrate the use of the chart, a set of conditions is assumed where the disc constant, or the number of watt-hours per revolution, is 10.0, and the product of the instrument transformer ratios is 30.0. Under these conditions, an observation is assumed to show a disc speed of 4.00 revolutions per minute.

First, a straight line is established through 10.0 on the *Meter Disc Constant* scale and 30.0 on the *Product of Instrument Transformer Ratios* scale, and a point is noted at the intersection of this line with the blank axis. Then, a second line is fixed by this point and the scale point for 4.00 on the *Disc R.P.M.* scale, and at its

intersection with the central scale, the load is read as 72.0 (the true value) on the *Average Kilowatts Load* scale and 96.6 on the *Average Horsepower*—(*Electrical*) scale, the computed value being 96.515.

For values of the variables beyond the scale limits, it need be remembered only that the variation is direct, and if it is necessary to multiply any one of the factors by 10, the final result is to be multiplied by 0.1, and similarly for other multipliers.

A key diagram on the chart should serve to obviate frequent reference to this text on repetition of use of the chart.



METER DISC.-RPM.
PRODUCT OF INST. TRANSFRMR. RATIOS
AVERAGE HORSEPOWER-(ELEC.)
AVERAGE KILOWATTS LOAD
PLANT DISC. CONSTANT
METER DISC. CONSTANT-WATT-HOURS/REV.

# 78. Power Factor and Total Power

**Method for the determination** of the total power in three-phase electrical circuits, as well as the power factor and phase angle, when two single-phase wattmeters are used. In this arrangement, each of the two instruments has its current coil connected into one of the phase conductors and its potential coil connected between its own phase conductor and the third conductor.

The total power in the system is the sum, algebraically, of the two instrument readings. At power factors below 50 per cent, one instrument must be reversed, since the indication is negative. The power factor is, of course, the cosine of the phase angle, and the expression showing this angle with regard to the readings of the two wattmeters is

$$\frac{W_1}{W_2} = \frac{\cos(30° - \phi)}{\cos(30° + \phi)}$$

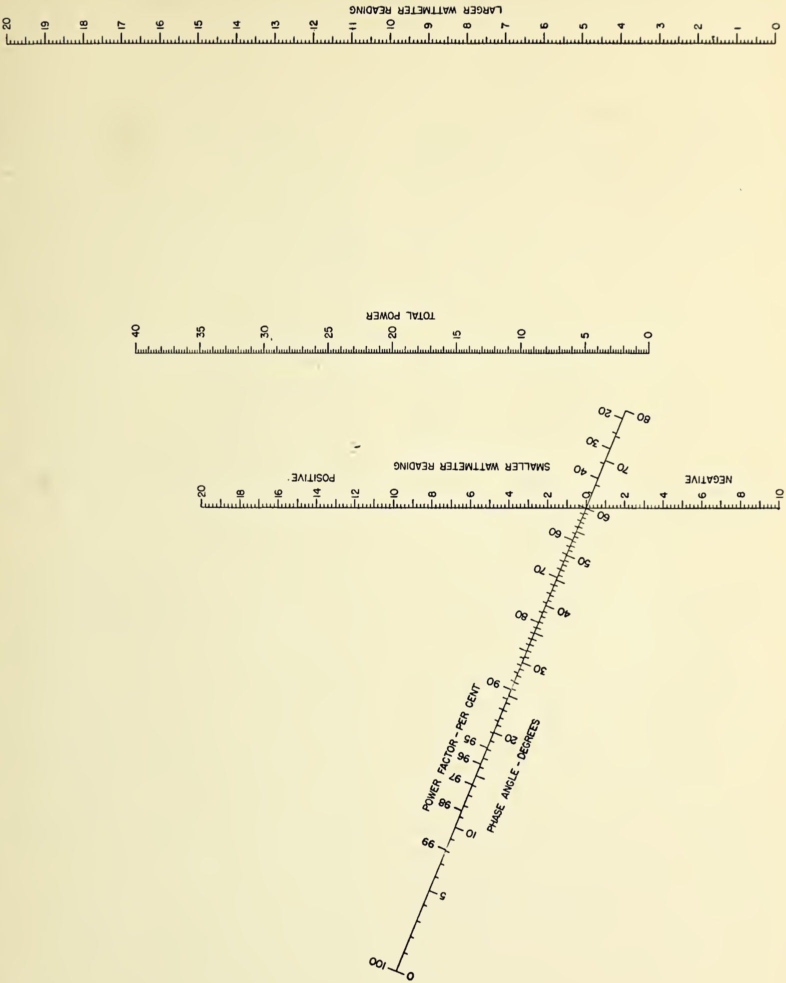
where $W_1$ is the larger of the two wattmeter readings, $W_2$ is the smaller, and $\phi$ is the phase angle, the power factor being then $\cos \phi$.

Naturally, the scale ranges cannot cover all possible cases, so in any set of observations the actual readings should be multiplied by some factor that will give results within the scale ranges, afterward multiplying the total power finding by the reciprocal of that same factor. If instrument transformers are used, the result so found must also be multiplied by the product of the ratios of current and potential transformers.

**Procedure.** To use the chart, a single operation is all that is required. This consists in establishing a straight line through the scale points corresponding to the values of *Larger Wattmeter Reading* and of *Smaller Wattmeter Reading* (using multipliers as necessary, in accordance with the remarks above). Then, at the intersection of this line with the *Total Power* scale is read that quantity, using multipliers when necessary, as explained. Also, at the intersection of this line with the sloping scale may be read the values of *Phase Angle* and *Power Factor*. These readings are actual values, and no adjustment by multipliers is necessary. Obviously, however, the same multipliers must be used for the readings of the two instruments.

To illustrate the use of the chart, it may be assumed that one instrument reads 150 watts and the other reads 50 watts. To bring these figures within the scale limits, a multiplier of 0.1 is used to give values of 15.0 and 5.00. Then, a straight line is established through the scale points for 15.0 on the *Larger Wattmeter Reading* scale and 5.00 on the *Smaller Wattmeter Reading* scale. At the intersection of this line with the sloping scale, the phase angle is read as 40.9° and the power factor is read as 75.5. Computation gives results of 40.91° and 75.58 per cent.

At the intersection of the established line with the *Total Power* scale is read the figure 20.0, and this must then be multiplied by the reciprocal of the multiplier originally used. So, the true total is 10.0 times the observed result, or 200. Then, if instrument transformers are used, this figure must be multiplied by the product of their ratios. Thus, if 4:1 potential transformers and 10:1 current transformers are used, the figure above must be multiplied by $4 \times 10$, or 40, and the final result is 8,000.


LARGER WATTMETER READING
TOTAL POWER
SMALLER WATTMETER READING
POSITIVE
NEGATIVE
POWER FACTOR - PER CENT
PHASE ANGLE - DEGREES

# 79. Power Factor and Reactive Power

**Method for the determination** of various characteristics of an alternating-current electrical load. It presents the relationship between the four elements (1) true power in kilowatts, the energy component of the electrical load, (2) the reactive power in kilovars, which is generally a magnetizing component, vectorially displaced 90° from the real component, (3) the kilovolt-ampere load on the circuit, the numerical product of current and voltage, without regard for vectorial angles, and (4) the power factor. This last item is the cosine of the phase angle as well as being the ratio of (1) to (3) above.

The mathematical basis for the relationship of items (1), (2), and (3) is that

$$\mathrm{KVA}^2 = \mathrm{KW}^2 + \mathrm{KVAR}^2$$

where KVA is the kilovolt-ampere product, KW is the value of true power, and KVAR is the reactive power, as described above.

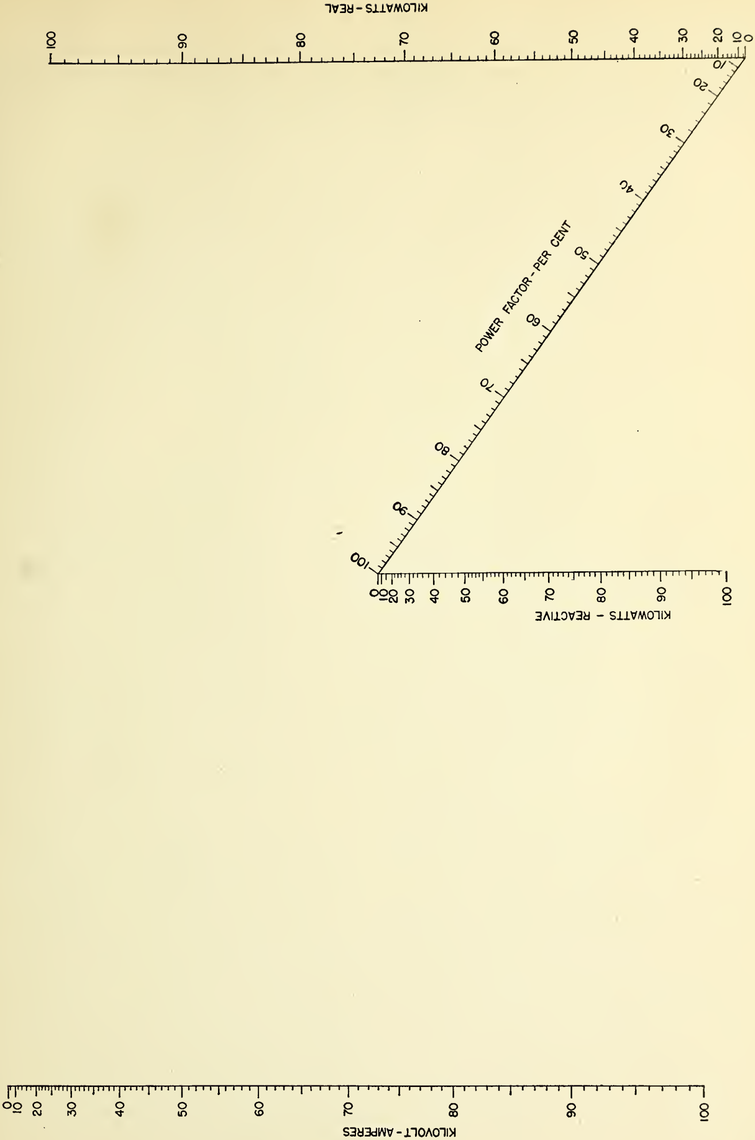
**Procedure.** To use the chart, it is necessary only to establish a straight line through the scale points corresponding to any two of the items (1) to (4) above. Then, at the intersection of this line with the other scales the missing quantities are read directly. The kilovolt-ampere component (3) is the simple product of current in amperes and potential in volts for a single-phase circuit, whereas it is this product multiplied in turn by the square root of 3, or 1.7321, for three-phase circuits.

Where the known figures are beyond the scale limits (except power factor), they should be multiplied by some factor which will bring the figures within the limits, later multiplying the results by the reciprocal of that factor.

To illustrate the use of the chart, a set of conditions is assumed where the wattmeter observation shows the real power to be 50.0 kilowatts, with the product of voltmeter and ammeter readings indicating 60.0 kilovolt-amperes. A straight line is established through the scale point of 50.0 on the *Kilowatts—Real* scale and that for 60.0 on the *Kilovolt-Amperes* scale. Then, at the intersection of this line with the *Power Factor* scale, that value is read as 83.3 per cent, and at its intersection with the *Kilowatts-Reactive* scale, this value is read as 33.3. The computed figures are 83.33 per cent and 33.17, respectively.

Taking a more complex operation, assume average voltmeter readings of 110.0, average ammeter readings of 4.00, and a wattmeter reading of 600, with a potential transformer ratio of 4:1 and a current transformer ratio of 50:1. True voltage is then 440, and current 200, with the wattage 120.0 kilowatts. Assuming also a three-phase circuit, the kilovolt-ampere value is $1.732 \times 440 \times 200/1{,}000$, or 152.42. Both kilowatt and kilovolt-ampere figures are beyond the scale limits, so it is suggested that they be multiplied by 0.5, giving figures of 60.0 and 76.21.

Then, a straight line is established through the scale point for 60.0 on the *Kilowatts—Real* scale and 76.21 on the *Kilovolt-Amperes* scale. At the intersection of this line with the *Power Factor* scale, the reading is 79.0 per cent (computed value


KILOWATTS - REAL
POWER FACTOR - PER CENT
KILOWATTS - REACTIVE
KILOVOLT - AMPERES

78.73), for which no correction is necessary. At the intersection of the same line with the *Kilowatts—Reactive* scale, the value is read as 46.7. But the basic figures were multiplied by 0.5 to come within scale limits, so this result must be multiplied by 2.0, becoming 93.4. The computed value is 94.0.

# 80. Power-factor Correction

**Method for the determination** of capacitor rating to bring the power factor of an electrical system from a known original condition to a desired improved condition. With utility rates often carrying penalties for low power factor and bonuses for high power factor, capacitor installations often repay their costs in but 2 or 3 years, and this chart will assist in arriving at optimum ratings of installations.

The capacitor rating necessary to bring the power factor of a system to another value is equal to the difference in the reactive power associated with the load under the two different conditions. The reactive power, KVAR, per kilowatt of real power in a system is equal to the tangent of the angle whose cosine is the power factor, this latter quantity being expressed as a decimal, rather than in per cent. These facts are pointed out as indicating the basis for the design of the chart, but they do not have to be considered in using the chart, since the scalings take these relationships into account.

**Procedure.** In using the chart, the first step is in the establishment of a straight line through the scale points corresponding to the values of known *Original Power Factor* and desired *Corrected Power Factor,* on their respective scales. Then, at the intersection of this line with the scale so designated is read the scale value for *Capacitor Requirement per Kilowatt.* Next, a second straight line is fixed by this point and the scale point corresponding to the known value of *Load—Kilowatts,* and the scale point of the intersection of this line with the *Capacitor Requirement* scale will give the required figure.

It should be noted that in connection with the second operation, double scaling appears on *Load—Kilowatts* and *Capacitor Requirement* scales, one, in each case, being marked *A* and the other *B*. These alternate scalings will serve to bring any values into portions of the scales which result in best conditions for accuracy. But when the *A* scale is used for one, the *A* scale must be used for the other, and likewise with the *B* scales.

Further, for values falling outside the scale limits, the actual load figure may be multiplied by any factor suitable for achieving a figure within the scale limits, afterward multiplying the result by the reciprocal of the original factor. This, together with the double-scaling feature, gives wide latitude in adjustment to obtain the greatest accuracy.

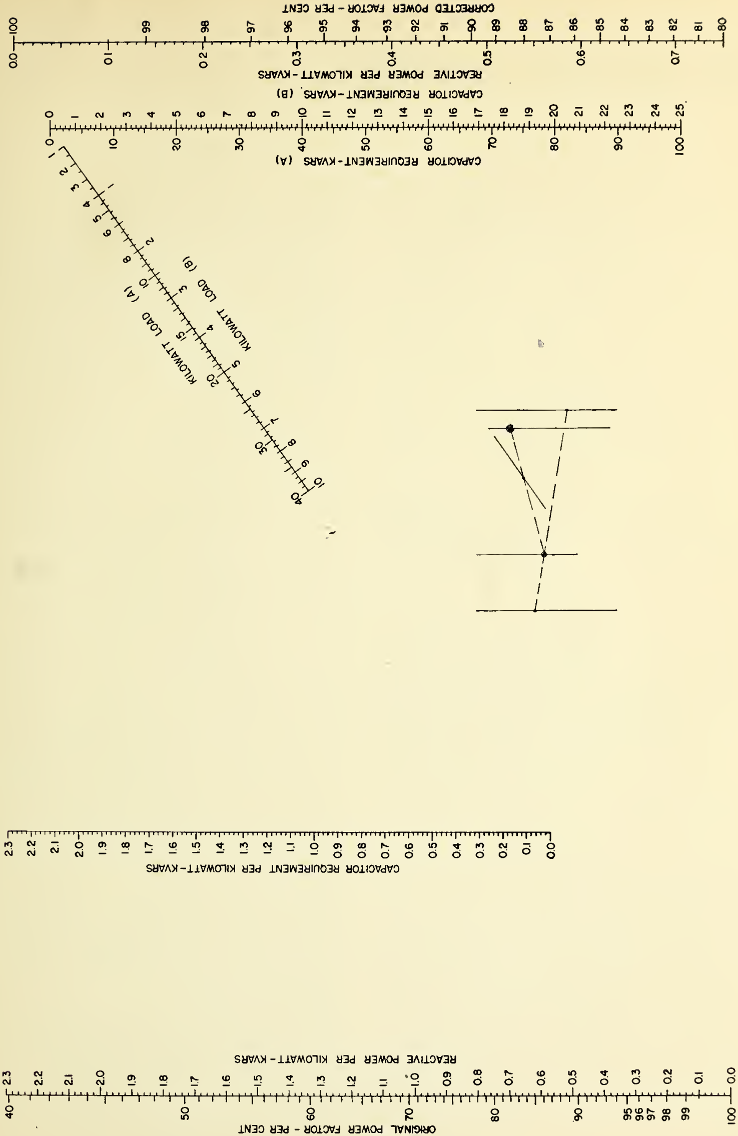
Thus, if the kilowatt load is 3.00, that scale point on the *A* scale could be used, but greater accuracy is possible by using the same value on the *B* scale. Or, the scale point for 30.0 on the *A* scale might be used, later dividing the figure for capacitor requirement by 10.

To illustrate the use of the chart, a set of conditions is assumed where the load is 10.00 kilowatts, the original power factor is 75.0 per cent, and it is desired to raise it to 95.0 per cent. First, a straight line is established through the scale points for 75.0 on the *Original Power Factor—Per Cent*

scale and 95.0 on the *Corrected Power Factor—Per Cent* scale, and at the intersection of this line with the *Capacitor Requirement per Kilowatt* scale is read the scale value 0.552 (computed value 0.5531).

Then, a second straight line is fixed by this point and the scale point corresponding to 10.0 on the *Load—Kilowatts* scale, and at the intersection of this line with the *Capacitor Requirement* scale this result is read, 5.55 when using the *A* scales and 5.50 when using the *B* scales, the computed value being 5.531.

A key diagram on the chart should serve as a guide in use of the chart, saving frequent reference to this text.


CORRECTED POWER FACTOR - PER CENT
REACTIVE POWER PER KILOWATT-KVARS
CAPACITOR REQUIREMENT-KVARS (B)
CAPACITOR REQUIREMENT-KVARS (A)
KILOWATT LOAD (B)
KILOWATT LOAD (A)
CAPACITOR REQUIREMENT PER KILOWATT-KVARS
REACTIVE POWER PER KILOWATT - KVARS
ORIGINAL POWER FACTOR - PER CENT

# 81. Equivalent Spacing

**Method for the determination** of the equivalent uniform spacing of the three conductors of a three-phase electrical system where the actual spacing is not uniform. Where computations are necessary on three-phase circuit characteristics, this is one of the basic items of information, and while the equation involved is simple, its solution requires a series of computations and reference to cube-root tables, all of which take far more time than is necessary with this chart.

The basic equation, as stated, is simple and is the one generally used. The equivalent spacing, in whatever units may be used, is

$$s = \sqrt[3]{abc}$$

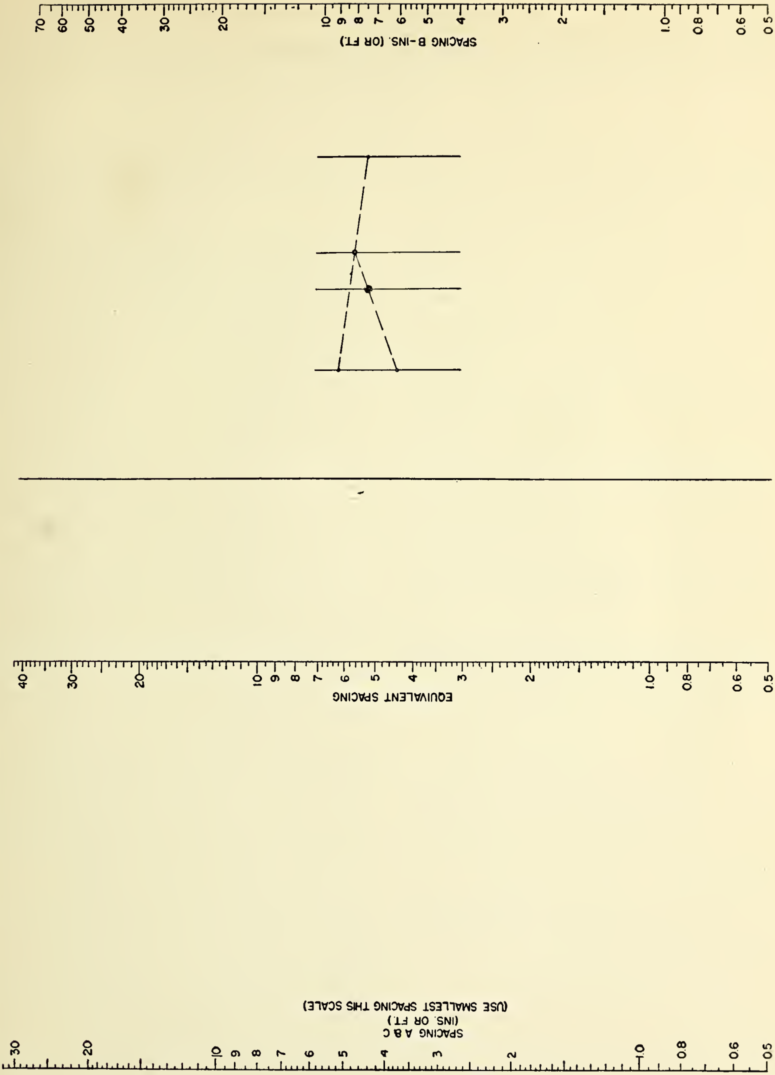
where $a$, $b$, and $c$ are the actual spacings between each combination of two of the three conductors. Naturally, the units must be consistent, *i.e.*, if the individual spacings are measured in inches, the equivalent spacing will be in the same unit. Obviously, all three individual spacings must be in the same units.

**Procedure.** In the chart, the scale limits are not identical, so the two smaller spacings may be designated as $A$ and $C$. Then, the first step in use of the chart is to establish a straight line through the scale points for $A$ and $B$ on the *Spacing A & C* and *Spacing B* scales, respectively. The intersection of this line is then noted on the blank axis. A second line is then fixed by this point and the scale point corresponding to the value of $C$ on the *Spacing A & C* scale. Then, at the intersection of this line with the *Equivalent Spacing* scale may be read the value for this quantity, in the units used for the individual spacings. The unit employed may be any measurement of length, even to a multiple of a basic unit, should this be necessary to bring the given values within the limits of the scalings.

To illustrate the use of the chart, a condition may be assumed wherein two conductors are carried at one level, at opposite sides of a pole near the ends of an arm, 11.0 feet apart, with the third conductor 8.0 feet higher, on the vertical center line between the other two. Simple computation, or the use of Chart 23, gives the distance between the upper conductor and each of the other two as 9.708 feet. The actual spacings are then 9.708 for $A$, 11.000 for $B$, and 9.708 for $C$.

Accordingly, a straight line is established through the scale point for 9.708 on the *Spacing A & C* scale and the scale point for 11.00 on the *Spacing B* scale, and the intersection of this line is noted on the blank axis. A second straight line is then fixed through this point and the scale point for 9.708 on the *Spacing A & C* scale (again). Then, at the intersection of this line with the *Equivalent Spacing* scale, the result is read as 10.15 feet, the computed value being 10.186 feet.

A key diagram on the chart should serve to avoid frequent reference to this text, by indicating the proper sequence of operations.



SPACING B-INS. (OR FT.)
EQUIVALENT SPACING
SPACING A B C
(INS. OR FT.)
(USE SMALLEST SPACING THIS SCALE)

# 82. Inductance and Reactance

**Method for the determination** of the self-inductance and reactance of each conductor of a single-phase, two-conductor line, or of each conductor of a three-phase, three-conductor line. Tables are available for these functions in any electrical engineers' handbook, but they have marked limitations. Interpolation is necessary more often than not and, if accurately done, is a time-consuming process. Further, to be complete, the table must be complete for each frequency, and rarely does coverage extend beyond 60- and possibly 25-cycle frequency.

This chart, however, is believed to cover all of the practical range of all variables, with accuracy equal to any tables, and, further, solution is possible with far greater ease and in a fraction of the time. The chart is based upon two equations, the first of which, for inductance, is to the effect that inductance in millihenrys per mile for each conductor is

$$L = 0.08047 + 0.74113 \log_{10} (2D/d)$$

where $D$ is the equivalent conductor spacing between centers and $d$ is the conductor diameter, both being in the same units of measurement.

The second equation is that for reactance, wherein the reactance in ohms per mile for each conductor is

$$X = 2\pi fL/1{,}000$$

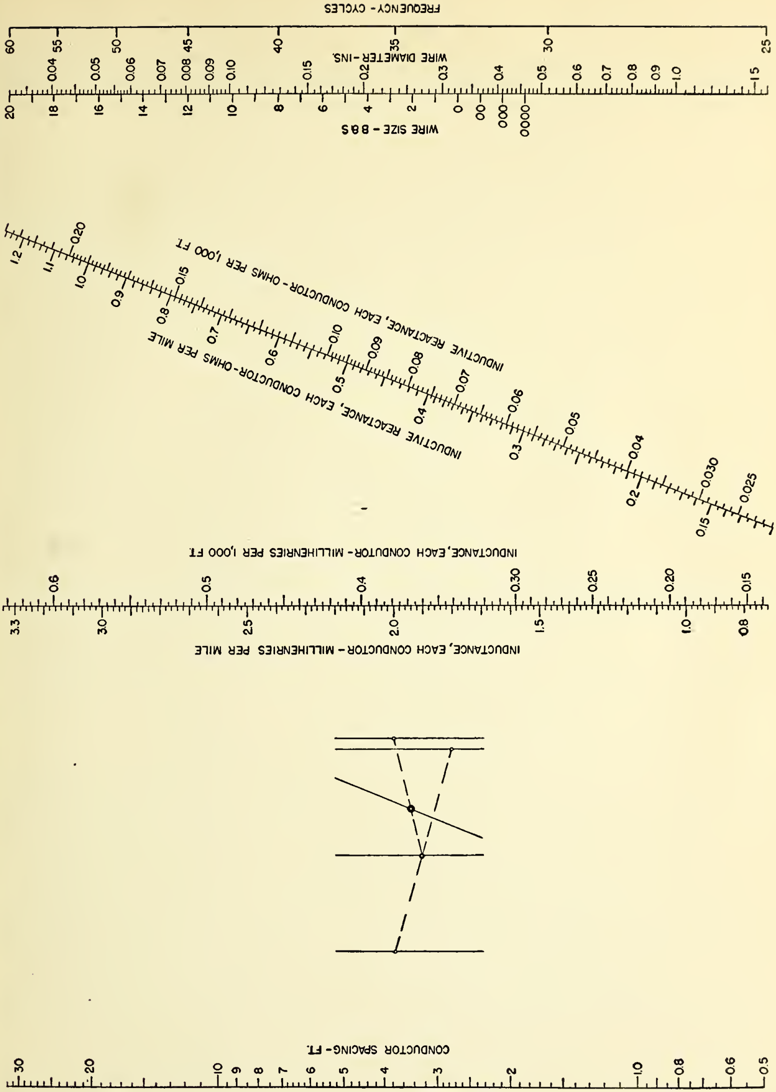
where $f$ is the system frequency in cycles per second and $L$ is as above. In the case of stranded conductors, $d$ is taken as the diameter of a solid round conductor of area equivalent to the stranded conductor.

**Procedure.** To use the chart, a straight line is established through the scale points corresponding to the known values of *Conductor Spacing* (equivalent) and *Conductor Diameter,* or *Wire Size,* on the scales so designated. Then, at the intersection of this line with the central vertical scale is read the *Inductance of Each Conductor,* in millihenrys, either per mile or per 1,000 feet.

A second straight line is then fixed by this point and the scale point corresponding to the known value for *Frequency,* on that scale. Then, at the intersection of that line with the sloping scale, the scale point indicates the value of *Inductive Reactance of Each Conductor,* in ohms, again either per mile or per 1,000 feet.

To illustrate the use of the chart, a case may be assumed where, the system frequency being 60 cycles, the equivalent spacing of the conductors is 10.15 feet, with a conductor diameter of 0.3648 inch, the size being No. 00 A.W.G. First, a straight line through the scale point for 10.15 on the *Conductor Spacing* scale and No. 00 *Wire Size* (or 0.3648 *Conductor Diameter*) on the appropriate scale is found to intersect the central vertical scale at a scaling indicating *Inductance of Each Conductor* as 2.175 millihenrys per mile. Actual computation gave a value of 2.172 millihenrys.

A second straight line is then fixed by the above point and the scale point corresponding to 60 on the *Frequency* scale.


FREQUENCY - CYCLES
WIRE DIAMETER - INS.
WIRE SIZE - B & S
INDUCTIVE REACTANCE, EACH CONDUCTOR - OHMS PER 1,000 FT
INDUCTIVE REACTANCE, EACH CONDUCTOR - OHMS PER MILE
INDUCTANCE, EACH CONDUTOR - MILLIHENRIES PER 1,000 FT
INDUCTANCE, EACH CONDUCTOR - MILLIHENRIES PER MILE
CONDUCTOR SPACING - FT.

Then, at the intersection of this line with the sloping scale, the scale points indicate *Inductive Reactance* as 0.820 ohm per mile, the computed value being 0.8200 ohm.

A key diagram on the chart should serve to avoid frequent reference to this text, by indicating the proper sequence of operations.

# 83. Capacitance and Susceptance

**Method for the determination** of the capacitance and susceptance between conductors in single-phase and three-phase transmission lines. Tables are available for these functions in any electrical engineers' handbook, but they have marked limitations. Interpolation is necessary more often than not and, if accurately done, is a time-consuming process. Further, to be complete, the tables must be complete for each frequency, and rarely does coverage extend beyond 60- and possibly 25-cycle frequency.

This chart, however, is believed to cover all of the practical range of all variables, with accuracy equal to any tables, and, further, solution is possible with far greater ease and in a fraction of the time. The chart is based upon two equations, the first of which, for capacitance, is to the effect that capacitance, in microfarads per mile, between conductors is

$$C = \frac{0.01941}{\log_{10}(2D/d)}$$

where $D$ is the equivalent conductor spacing between centers and $d$ is the equivalent conductor diameter. For solid conductors, this latter is the actual diameter, whereas for stranded conductors it is the outside diameter of the stranded cable itself. Both $D$ and $d$ must be in the same units.

The second equation is that for susceptance, wherein the susceptance, in micromhos per mile, between conductors is

$$X = 2\pi f C$$

where $f$ is the system frequency in cycles per second.

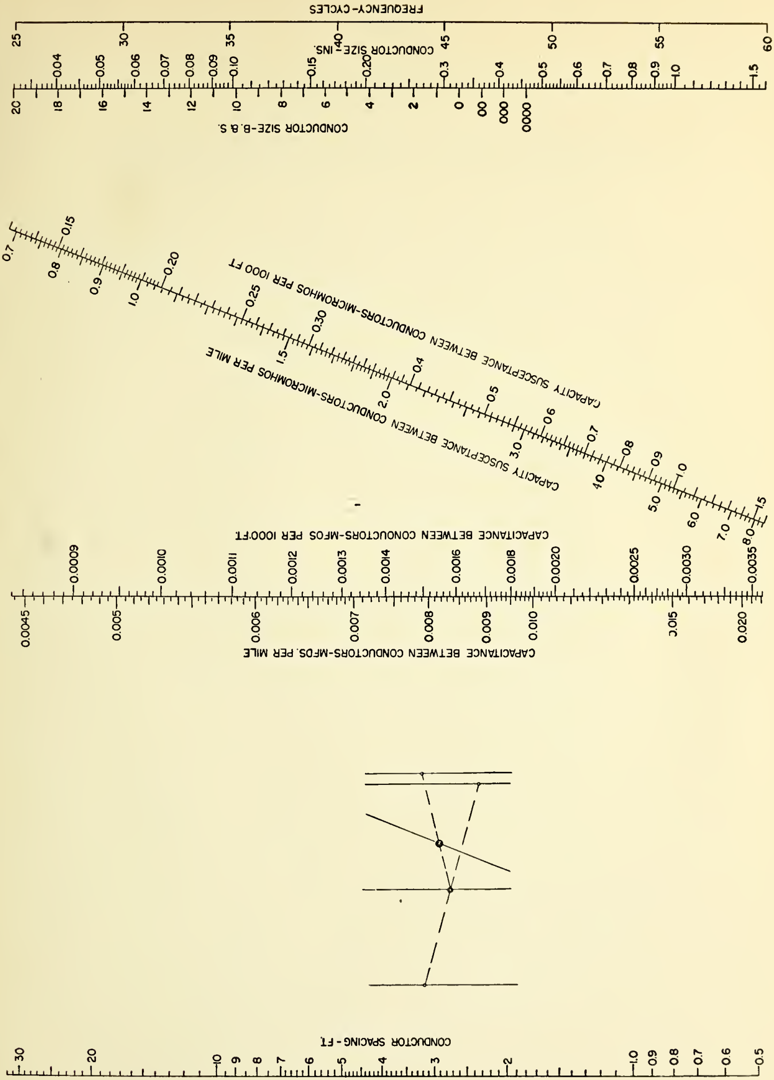
**Procedure.** To use the chart, a straight line is established through the scale points corresponding to the known values of *Conductor Spacing* (equivalent) and *Conductor Diameter,* or *Wire Size,* on the scales so designated. Then, at the intersection of this line with the central vertical scale is read the *Capacitance between Conductors,* in microfarads per mile or per 1,000 feet. Note that capacitance to neutral, a quantity often required, is *twice* the above figure.

A second straight line is then fixed by this point and the scale point corresponding to the known value for *Frequency,* on that scale. Then, at the intersection of this line with the sloping axis, the scale point indicates the value of *Capacity Susceptance between Conductors* in micromhos per mile or per 1,000 feet. Again, to determine the value to neutral, the observed figure must be doubled.

To illustrate the use of the chart, a case may be assumed where, the system frequency being 60 cycles, the equivalent spacing of the conductors is 10.15 feet, with a conductor diameter of 0.4180 inch, the size being No. 00 A.W.G., stranded. First, a straight line is established through the scale points for 10.15 on the *Conductor Spacing* scale and for No. 00 *Wire Size* (or 0.4180 *Conductor Diameter*) on the appropriate scale, and this is found to intersect the central vertical scale at a point indicating *Capacitance between Conductors* as 0.00702 microfarad per mile. Computed value is 0.007014 microfarad per mile.

A second straight line is then fixed by this point and the scale point for 60 on the *Frequency* scale. Then, at the intersection of this line with the sloping axis, the scale points indicate *Capacity Susceptance* as 2.650 micromhos per mile. Actual computation gave a value of 2.6442 micromhos per mile.

A key diagram on the chart should serve to avoid frequent reference to this text, by indicating the proper sequence of operations.


FREQUENCY-CYCLES
CONDUCTOR SIZE-INS.
CONDUCTOR SIZE-B. & S.
CAPACITY SUSCEPTANCE BETWEEN CONDUCTORS-MICROMHOS PER 1000 FT
CAPACITY SUSCEPTANCE BETWEEN CONDUCTORS-MICROMHOS PER MILE
CAPACITANCE BETWEEN CONDUCTORS-MFDS PER 1000 FT
CAPACITANCE BETWEEN CONDUCTORS-MFDS. PER MILE
CONDUCTOR SPACING - FT.

# Charts 84 to 92

# TRANSMISSION-LINE CHARACTERISTICS

It is a long and tedious process to derive from known conditions of voltage and current at the delivery end of a transmission line, the voltage and current at the generator end. This is true even when recourse is had to one of the various approximate methods, solving equations based upon assumptions of lumping or localizing the actually uniformly distributed capacitance effect. But in some cases, such assumed localization of the capacitance may be the source of errors of considerable magnitude.

On the other hand, the use of the exact mathematical method of solution, based upon the integration of capacitance effect as a uniformly distributed effect, is a chore that involves many hours of computation to determine even the first set of conditions. True, there are available various graphic methods for solution of this problem, but most are based upon one or another of the various approximate methods of solution, so the saving of time may result in a large degree of inaccuracy.

Of course, there are cases where this inaccuracy is of little importance. If the line is short, the errors introduced by approximate methods are so small as to be insignificant. Further, the conditions under which the line is operated may make such errors of little importance.

For such cases as these, the three charts immediately preceding, in combination with any charts or tabulations based upon approximate methods, may give solutions that are completely satisfactory and adequate. But on *any* long lines, or on others where close prediction of characteristics is important, the errors introduced by the approximate methods of solution may be of serious import, and solution by such methods may lead to serious difficulties.

In the following group of charts, however, solutions are possible with little expenditure of time, but with little sacrifice of accuracy, and that only to the extent of interpolation on scales, which, having to cover wide ranges, may not give results to as many significant figures as the actual computations would. But then, the fact remains that the basic data for solution of the problem cannot, ordinarily, be any more accurate, since, for example, even the resistance of conductors, with variations in temperature, will vary from mile to mile along the length of the line.

Certainly, there will be no introduction of error by reason of use of approximate and possibly inaccurate equations. For the equations upon which the charts are based are the exact equations as arrived at by setting up differential equations and then integrating. Most of the factors in these equations are, of course, vector or complex quantities, and the charts give a step-by-step solution of those equations, which are

$$E_1 = E_2 \cosh \sqrt{ZY}\, l + I_2\sqrt{Z/Y} \sinh \sqrt{ZY}\, l \quad (1)$$

and

$$I_1 = I_2 \cosh \sqrt{ZY}\, l + E_2\sqrt{Y/Z} \sinh \sqrt{ZY}\, l \quad (2)$$

where $E_1$ and $I_1$ are, respectively, voltage and current conditions at the generator end of the line, $E_2$ and $I_2$ are the corresponding conditions at the delivery end, $l$ is the length of the line, in any length units, $Z$ is the line impedance in ohms per unit of line length, and $Y$ is the line susceptance in mhos per unit of line length.

In the designation and evaluation of the various vectorial or complex factors, the value of the delivery-end voltage $E_2$ is assumed as the angularly basic factor, *i.e.*, the vector angle of this factor is taken as zero.

To simplify the equations, the various functions of $Y$ and $Z$ will be represented hereafter by other symbols, $V$, $U$, and $W$, representing, respectively, $\sqrt{ZY}$, $\sqrt{Z/Y}$, and $\sqrt{Y/Z}$. The equations then become

$$E_1 = E_2 \cosh Vl + I_2U \sinh Vl \quad (3)$$

and

$$I_1 = I_2 \cosh Vl + E_2W \sinh Vl \quad (4)$$

It is obvious that, with the exception of the voltage and current terms in these equations, all of the factors are in the nature of line constants, being fixed by the design of the line itself. Consequently, after these line-constant factors have once been evaluated for a given line, solutions for various conditions of load will require the use of only a few of the charts in this group.

To point up this resolving of the equations, they may now be simplified further, introducing new symbols for these fixed line constants. Accordingly $A$ is taken as representing cosh $Vl$, $B$ is substituted for $U$ sinh $Vl$, and $D$ is used in place of $W$ sinh $Vl$. The equations are now reduced to the form

$$E_1 = E_2A + I_2B \quad (5)$$

and

$$I_1 = I_2A + E_2D \quad (6)$$

Thus, once the line constants have been determined, the solutions for various load conditions are simple. They consist only, in each case, in evaluating the terms on the right-hand side of the equations above, by the use of charts, making the simple double additions, and then, by means of the last chart of the group, transforming them back to vectorial quantities, *i.e.*, quantities with fixed magnitude and phase angle with respect to the base, which is delivery voltage.

# 84. Hyperbolic Angle

**Method for the determination** of the transmission-line characteristic known as the hyperbolic angle, or propagation constant. Specifically, it is the square root of the product of impedance and admittance, both of which are vector quantities. These quantities, then, are measured in direction as well as magnitude, having both real and imaginary components, and the chart is designed to handle these complex quantities.

Chart 83 gives a method for the determination of susceptance, usually assumed as equivalent to admittance, and its vectorial angle is accordingly taken as 90°, from which any departure will be negligible. Chart 82 provides a means for determination of inductive reactance, and recourse to standard tables provides the value for resistance, the other component of impedance. Impedance, of course, is the square root of the sum of the squares of resistance and inductive reactance, as to numerical value.

**Procedure.** The first step in the use of the chart is the determination of the vector quantity impedance. This is done by locating in the intersection-type-nomograph section at the left-hand side of the chart, the point of intersection of the curves of known values for *Inductive Reactance* and *Resistance,* interpolating as necessary. Then, following upward and to the left along the sloping lines, the intercept of this point is noted on the scaled line marking the left-hand edge of this section, and the scaling at that point indicates the numerical magnitude of the *Impedance.*

From the curve-intersection point, a vertical line is followed downward to the lower edge of this section of the chart, and the intercept so located will be at a scale point corresponding to the *Impedance—Phase Angle.* Thus, the impedance is evaluated as a vector quantity by magnitude and phase angle.

Now, the hyperbolic angle, or propagation constant, is also a vectorial quantity, and *it* must be evaluated as to magnitude and angle. To determine its magnitude, a straight line is established through the scale points corresponding to the magnitudes of *Impedance* and *Capacity Susceptance* on the scales so designated. Then, at the intersection of this line with the *Hyperbolic Angle* scale, the magnitude of this factor is read.

To determine the vector angle of this quantity, a second straight line is established through the scale point previously located on the *Impedance—Phase Angle* scale and the scale point noted 90° at the lower end of the *Capacity Susceptance* scale. Then, at the intersection of this line with the *Phase Angle* scale of the central section of the chart is read the vector angle of the *Hyperbolic Angle,* or propagation constant.

The next step in a transmission-line problem requires that this vector quantity be resolved into its real and imaginary components, or, otherwise described, the in-phase and quadrature components. This is accomplished by following upward along the vertical line corresponding to the phase angle to its intersection with the sloping

line corresponding to the previously located scale point for the magnitude of this quantity. Then this point in relation to the curves for *In-phase Component* and *Quadrature Component,* interpolating as necessary, gives those values.

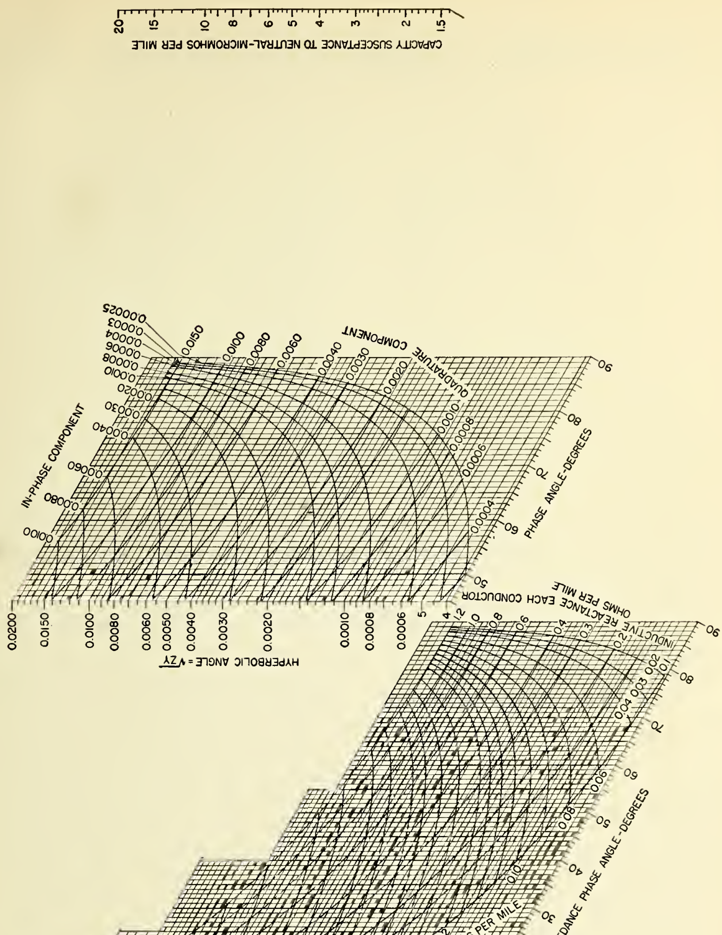
Ordinarily, resistance, reactance, and susceptance are determined per mile of line length, and the chart is designed to cover the normal variation of those quantities for such a unit. For further computations on a transmission line, however, the total line value for hyperbolic angle, or propagation constant, must be known. To evaluate this, all that is necessary is a simple arithmetical computation, the multiplication of each of the components per unit length by the value of line length in those same units.

To illustrate the use of the chart, a set of conditions may be assumed as in Charts 81, 82, and 83, wherein, with a 60-cycle frequency, a three-phase system of No. 00 A.W.G. conductors has an equivalent spacing of 10.15 feet. From those charts, the inductive reactance per mile has been determined as 0.820 ohm and the capacity susceptance per mile (to neutral) has been found to be 5.30 micromhos per mile. Reference to wire tables gives the resistance as 0.428 ohm per mile.

The first step is to evaluate the impedance as to magnitude and angle. So, a point is located in the left-hand section of the chart (interpolating between curves) corresponding to 0.820 ohm *Inductive Reactance* and 0.428 ohm *Resistance.* Following a line vertically downward from this point, the *Impedance—Phase Angle* is read as 63° (computed value 62° 26′). Following a sloping line upward to the left indicates the *Impedance—Magnitude* as 0.925 (computed value being 0.9250).

Now, to find the magnitude of the hyperbolic-angle vector, a straight line is established through the scale point found for the magnitude of *Impedance* and the scale point for 5.30 on the *Capacity Susceptance* scale. This line is found to intersect the *Hyperbolic Angle* scale at a scale point indicating 0.00223 (computed value being 0.002211). Another line is then fixed by the previously determined scale point (63°) on the *Impedance—Phase Angle* scale and the 90° scale point at the lower end of the *Capacity Susceptance* scale. Then, at the intersection of this line with the *Hyperbolic Angle—Phase Angle* scale, the vector angle is read as 76° 30′ (computed value being 76° 13′).

Then, following along the vertical and sloping lines of this section, the intersection is located and evaluated by interpolation between the curves. The *In-phase Component* is then determined as 0.00052, and the *Quadrature Component* is found to be 0.00220. Computation gives values of 0.0005267 and 0.002147, respectively.


CAPACITY SUSCEPTANCE TO NEUTRAL-MICROMHOS PER MILE
HYPERBOLIC ANGLE = √ZY
IN-PHASE COMPONENT
QUADRATURE COMPONENT
PHASE ANGLE-DEGREES
INDUCTIVE REACTANCE EACH CONDUCTOR OHMS PER MILE
RESISTANCE EACH CONDUCTOR-OHMS PER MILE
IMPEDANCE PHASE ANGLE-DEGREES
IMPEDANCE EACH CONDUCTOR-OHMS PER MILE

# 85. Hyperbolic Cosine of Hyperbolic Angle

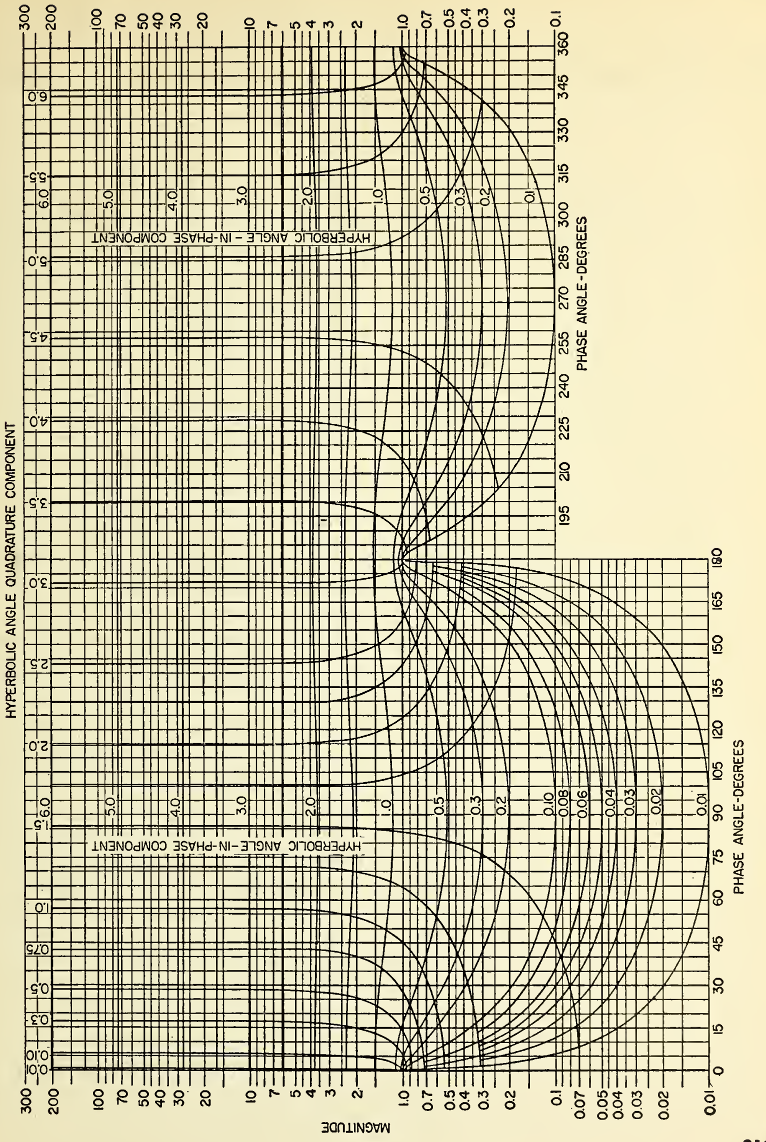
**Method for the determination** of one of the line constants necessary to be known in the solution of problems of transmission-line characteristics. Specifically, it serves to evaluate the term $A$ in the simplified forms of Eqs. (5) and (6) on page 214. The derivations of the forms of these equations have been explained there in a general discussion of the transmission-line charts immediately preceding Chart 84.

By means of Chart 84, the so-called hyperbolic angle, or propagation constant, may be determined per unit length of line. But it is necessary to evaluate this factor as a function of the entire line and, as described on page 216, this involves a simple multiplication of the in-phase and quadrature components of this factor per mile by the line length in miles. This operation then gives an in-phase and a quadrature component of the hyperbolic angle, or propagation constant, for the line as a whole.

**Procedure.** To use the chart, a point is located at the intersection of the curve corresponding to the computed value of *Hyperbolic Angle—In-phase Component,* and that of the computed value of *Hyperbolic Angle—Quadrature Component,* interpolating as necessary. Then, the scalings of the rectangular-coordinate system give values for the *Magnitude* and *Phase Angle* of the *Hyperbolic Cosine of the Hyperbolic Angle.* This corresponds to the terms cosh $\sqrt{ZY}\,l$ of Eqs. (1) and (2), cosh $Vl$ of Eqs. (3) and (4), and $A$ of Eqs. (5) and (6).

To illustrate the use of the chart, the same set of conditions is assumed as in preceding charts. From Chart 84, the in-phase and quadrature components of the hyperbolic angle were determined as 0.00052 and 0.00220, respectively, per mile. Assuming a line length of 150 miles, the values for the line as a whole are 0.0780 and 0.330, respectively.

Then, the point is located on the chart, by interpolation, corresponding to the intersection of a curve value of 0.078 for *Hyperbolic Angle—In-phase Component* and a curve value of 0.330 for *Hyperbolic Angle—Quadrature Component.* The intercepts of this point on the rectangular-coordinate system are at scale values corresponding to 0.96 for *Magnitude* and 1° for *Phase Angle* of the *Hyperbolic Cosine of the Hyperbolic Angle.* Actual computation gave figures of 0.9518 and 1° 21.1′.


HYPERBOLIC ANGLE QUADRATURE COMPONENT
HYPERBOLIC ANGLE - IN-PHASE COMPONENT
HYPERBOLIC ANGLE-IN-PHASE COMPONENT
PHASE ANGLE-DEGREES
PHASE ANGLE - DEGREES
MAGNITUDE

# 86. Surge Impedance

**Method for the determination** of one of the line constants required to be known in predicting the voltage characteristics of a transmission line. Specifically, surge impedance is the square root of the quantity obtained by the division of the value of impedance per unit length of line by the admittance (equal to capacity susceptance) per unit length, both being vector quantities. In the general discussion preceding Chart 84, it appears as $\sqrt{Z/Y}$ in Eq. (1) and as $U$ in Eq. (3).

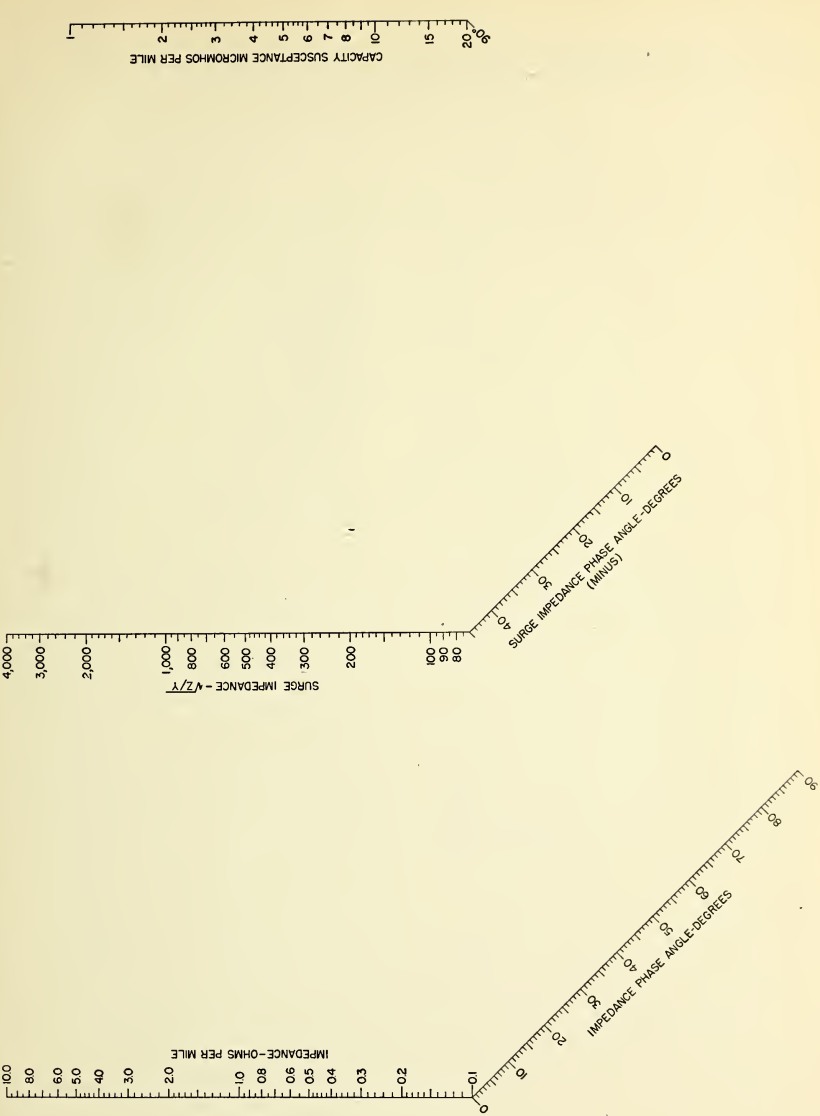
In Chart 84 are means for evaluating the impedance as a vector quantity from known values of resistance and inductive reactance, and the magnitude and phase angle of this quantity are used again in this chart. Capacity susceptance may be determined from Chart 83, being also a vector quantity, but with phase angle assumed as 90° from the base of angle measurement, here, as usual, taken as the phase angle of the delivery voltage of the system.

**Procedure.** To use the chart, two steps are necessary: one to determine the magnitude, and the second to determine the phase angle of the surge impedance. For the former, a straight line is established through the scale points corresponding to the known values of *Impedance in Ohms per Mile* and *Capacity Susceptance in Micromhos per Mile,* on the scales so designated. Then at the intersection of this line with the *Surge Impedance* scale, the magnitude of this quantity is read on the scale.

For the second step, a second straight line is established through the scale points corresponding to the known values of *Impedance—Phase Angle* and of *Capacity Susceptance—Phase Angle,* the latter being the point noted as 90° and located at the lower end of the *Capacity Susceptance* scale. Then at the intersection of this line with the *Surge Impedance—Phase Angle* scale is read this quantity on the scale, noting that this will always have a negative value. These two operations then completely evaluate the surge impedance as a vector quantity, both magnitude and phase angle being known.

To illustrate the use of the chart, the same set of data is assumed as was used in the charts immediately preceding. Impedance was determined from Chart 84 as 0.925 ohm per mile with a phase angle of 63°. Capacity susceptance as determined by Chart 83 was 2.65 micromhos per mile between conductors, or 5.30 to neutral, the phase angle being accepted as 90°.

First, a straight line is established through the scale points for 0.925 on the *Impedance* scale and 5.30 on the *Capacity Susceptance* scale, and this is found to intersect the *Surge Impedance* scale at a scale point indicating the magnitude of the vector to be 420. A second straight line is then fixed by the scale points of 63° on the *Impedance—Phase Angle* scale and the 90° scale point on the *Capacity Susceptance* scale. This line is found to intersect the *Surge Impedance—Phase Angle* scale at a scale point indicating the vector angle as −13° 50′ (negative). Computation determined the vector quantity as having magnitude 418.22 and phase angle −13° 46.85′.


CAPACITY SUSCEPTANCE MICROMHOS PER MILE
SURGE IMPEDANCE PHASE ANGLE-DEGREES (MINUS)
SURGE IMPEDANCE - √Z/Y
IMPEDANCE PHASE ANGLE-DEGREES
IMPEDANCE-OHMS PER MILE

# 87. Hyperbolic Sine of Hyperbolic Angle

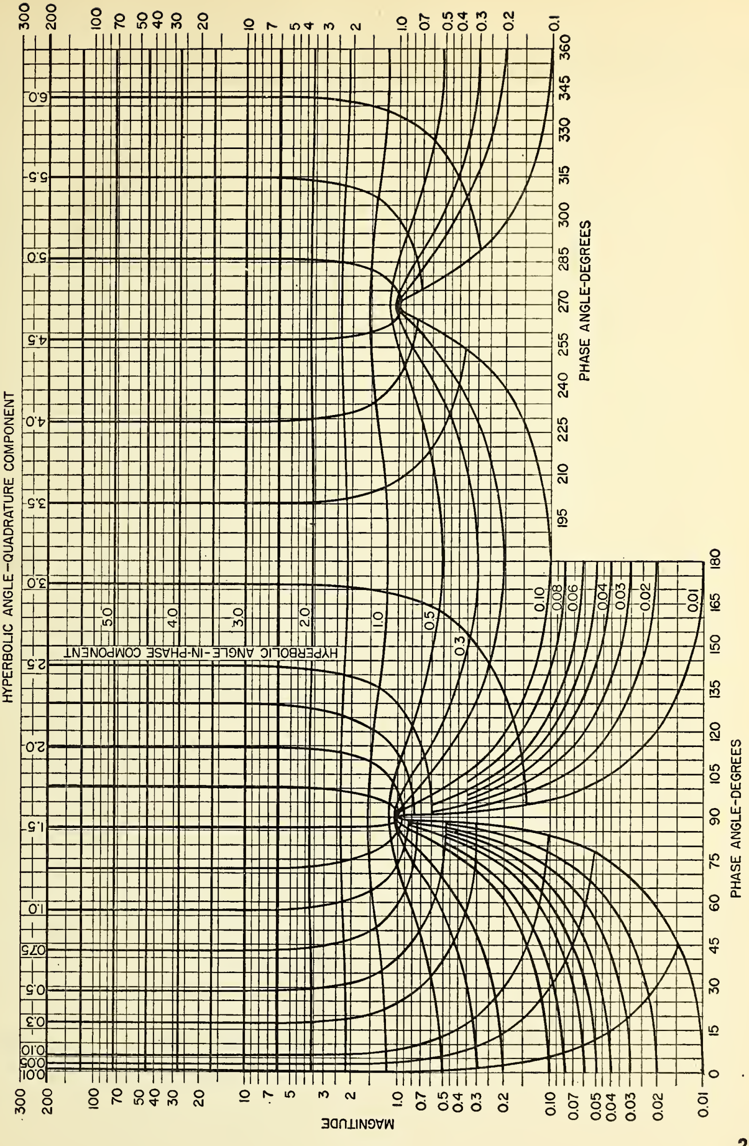
**Method for the determination** of one of the line constants necessary to be known in the solution of problems of transmission-line characteristics. Specifically, it serves to evaluate the term sinh $Vl$ in Eqs. (1), (2), (3), and (4), as referred to in the general discussion of the solution of transmission-line problems immediately preceding Chart 84, pages 213 and 214.

By means of Chart 84, the so-called hyperbolic angle, or propagation constant [term $V$ in Eqs. (3) and (4) on page 214], may be determined per unit length of line. Then, as described in the text accompanying Chart 85, the components of this factor *per unit length of line* must be multiplied by the length of line in the same units to obtain the value of those components for the line as a whole (page 216).

**Procedure.** To use the chart, a point is located at the intersection of the curve corresponding to the computed value of *Hyperbolic Angle—In-phase Component,* and that of the computed value of *Hyperbolic Angle—Quadrature Component,* interpolating as necessary. Then, the scalings of the rectangular-coordinate system give values for the *Magnitude* and *Phase Angle* of the *Hyperbolic Sine of the Hyperbolic Angle.* This corresponds to the terms sinh $\sqrt{ZY}\, l$ of Eqs. (1) and (2) and sinh $Vl$ of Eqs. (3) and (4), still referring to the text immediately preceding Chart 84.

To illustrate the use of the chart, the same set of conditions is assumed as in the preceding charts. From Chart 84, with the simple multiplication to bring its results to apply to a line length of 150 miles, the in-phase and quadrature components were determined as 0.0780 and 0.330, respectively.

Then, the point is located on the chart, by interpolation, corresponding to a curve value of 0.078 for *Hyperbolic Angle—In-phase Component,* and a curve value of 0.330 for *Hyperbolic Angle—Quadrature Component.* The intercepts of this point on the rectangular coordinate system are at scale values corresponding to 0.32 for *Magnitude* and 76.3° for *Phase Angle* of the *Hyperbolic Sine of the Hyperbolic Angle.* Actual computation gave figures of 0.32622 and 76° 42.2′.


HYPERBOLIC ANGLE-QUADRATURE COMPONENT
HYPERBOLIC ANGLE-IN-PHASE COMPONENT
PHASE ANGLE-DEGREES
PHASE ANGLE-DEGREES
MAGNITUDE

# 88. Surge Admittance

**Method for the determination** of one of the line constants required to be known in predicting the operating characteristics of a transmission line. Specifically, the surge admittance is the square root of the quantity obtained by the division of the value of capacity admittance (equal to capacity susceptance) per unit length of line, by the value of impedance per unit length, both being vector quantities. In the general discussion preceding Chart 84, it appears as $\sqrt{Y/Z}$ in Eq. (2) and as $W$ in Eq. (4).

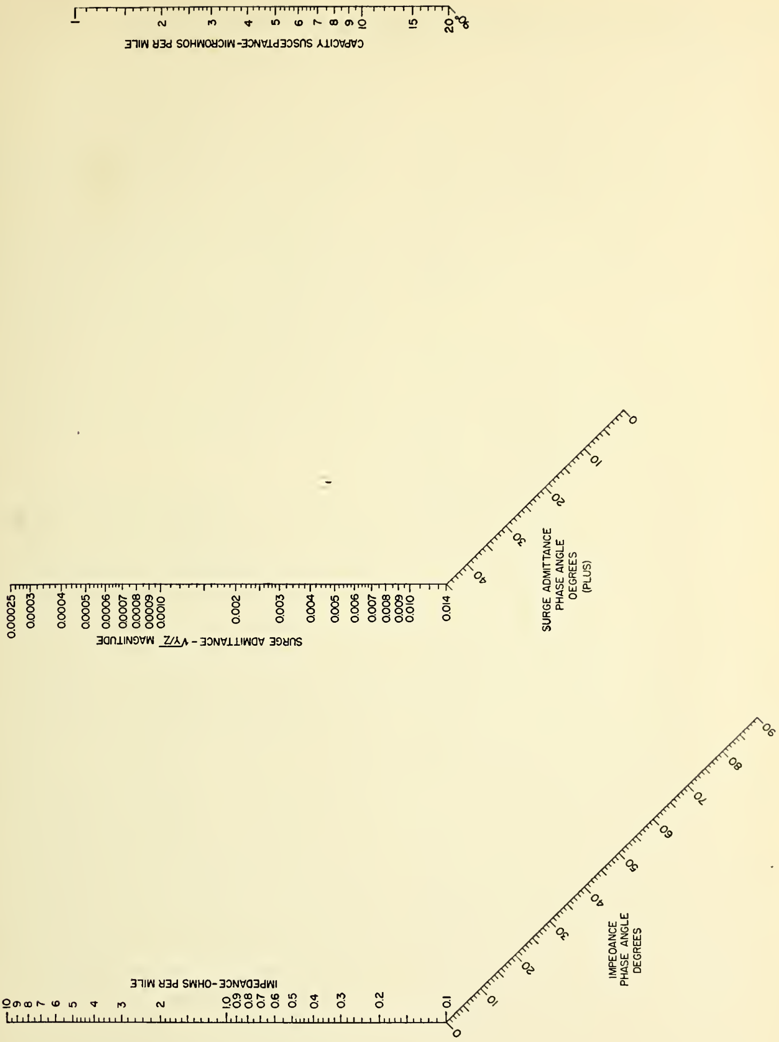
In Chart 84 are means for evaluating the impedance as a vector quantity from known values of resistance and inductive reactance, and the magnitude and phase angle of this quantity are used again in this chart. Capacity susceptance may be determined from Chart 83, being also a vector quantity, but with phase angle assumed at 90° from the base of angle measurement, here, as usual, taken as the phase angle of the delivery voltage of the system.

**Procedure.** To use the chart, two steps are necessary: one to determine the magnitude, and the second to determine the phase angle of the surge admittance. For the former, a straight line is established through the scale points corresponding to the known values of *Impedance—Ohms per Mile* and *Capacity Susceptance—Micromhos per Mile,* on the scales so designated. Then at the intersection of this line with the *Surge Admittance* scale, the magnitude of this quantity is read on the scaling there.

For the second step, a second straight line is established through the scale points corresponding to the known values of *Impedance—Phase Angle* and of *Capacity Susceptance—Phase Angle,* the latter being the point noted as 90° and located at the lower end of the *Capacity Susceptance* scale. Then at the intersection of this line with the *Surge Admittance—Phase Angle* scale is read this quantity on that scale, noting that the value is positive. These two operations then completely evaluate the surge admittance as a vector quantity, both magnitude and phase angle having been determined.

To illustrate the use of the chart, the same set of data is assumed as was used in the charts immediately preceding. Impedance was determined from Chart 84 as 0.925 ohm per mile with a phase angle of 63°. Capacity susceptance as determined by Chart 83 was 2.65 micromhos per mile between conductors, or 5.30 to neutral, the phase angle being accepted as 90°.

First, a straight line is established through the scale points for 0.925 on the *Impedance* scale and 5.30 on the *Capacity Susceptance* scale, and this is found to intersect the *Surge Admittance* scale at a scale point indicating the magnitude of the vector quantity as 0.00238. A second straight line is then fixed by the scale points of 63° on the *Impedance—Phase Angle* scale and the 90° scale point on the *Capacity Susceptance* scale. This line is found to intersect the *Surge Admittance—Phase Angle* scale at a scale point indicating the vector angle as 13° 45′. Computation determined the vector quantity as having magnitude 0.002391 and phase angle 13° 46.85′.


CAPACITY SUSCEPTANCE-MICROMHOS PER MILE
SURGE ADMITTANCE - $\sqrt{Y/Z}$ MAGNITUDE
SURGE ADMITTANCE PHASE ANGLE DEGREES (PLUS)
IMPEDANCE-OHMS PER MILE
IMPEDANCE PHASE ANGLE DEGREES

# 89. Voltage-characteristics Evaluation

**Method for the determination** of the line constants that enter into the solution of the equation for generator voltage on a transmission line where the load conditions are known. It continues the series on transmission-line characteristics, and reference is made to the section of text immediately preceding Chart 84.

In that general discussion of the solution of transmission lines, equations were presented in progressively simpler forms, for voltage and current characteristics, and this chart provides for the evaluation of the factors of the former, Eqs. (1), (3), and (5) on page 214. To save the need for referring back to those pages, Eqs. (3) and (5) are, in their simplified forms,

$$E_1 = E_2 \cosh Vl + I_2U \sinh Vl \qquad (3)$$

$$E_1 = E_2A + I_2B \qquad (5)$$

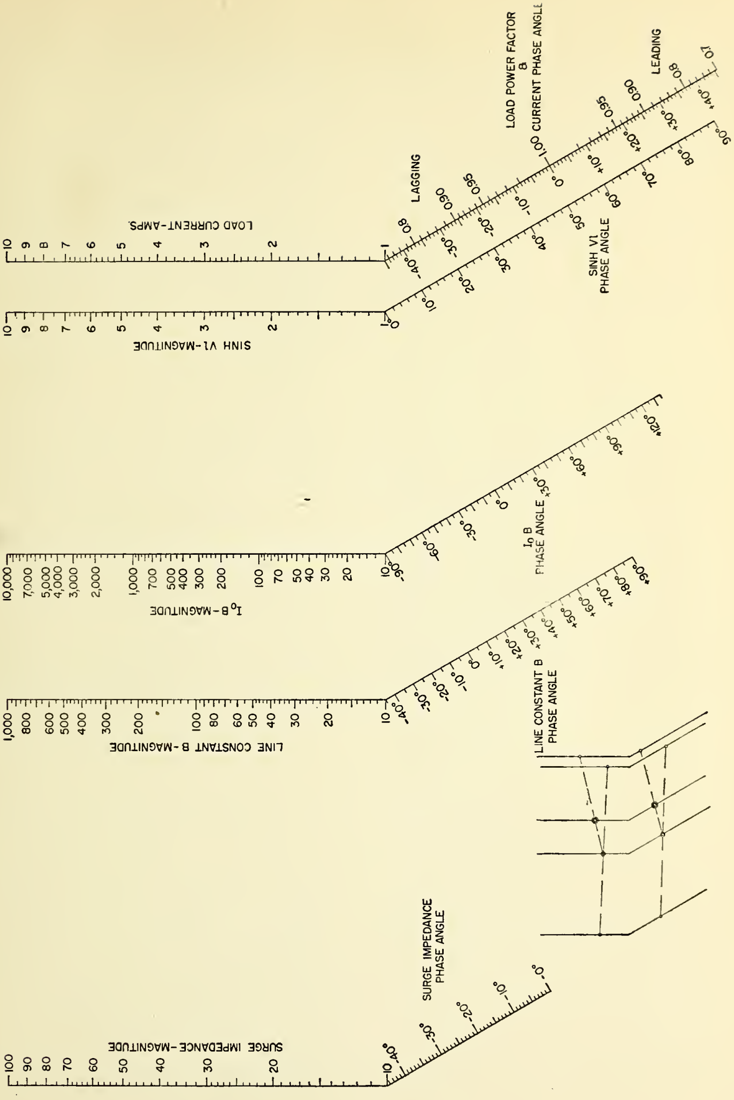
$A$ and $B$ are line constants, which may be directly evaluated, or whose factors may be evaluated from preceding charts of this group. All factors, of course, are vector quantities, fixed by magnitude and angular position, as related to the vector angle of delivery voltage as the zero.

As to the first term on the right-hand side of the equation, it was felt not worth while to include a chart for the multiplication of the two factors that make it up, since it requires only a simple arithmetical computation. The factor $E_2$, the delivery voltage, is simply used as a multiplier with the term $A$, or cosh $Vl$, as evaluated by means of Chart 85. The magnitude of the resulting term is the product of this voltage and the magnitude of the vector value of cosh $Vl$, while the vector angle of this product is equal to the vector angle of cosh $Vl$, or $A$.

This chart now handles the multiplication of the three factors of the second term on the right-hand side of the equation, all three factors being vector quantities, being fixed, in each case, in magnitude and direction, or angle. The factor $U$, or surge impedance, $(\sqrt{Z/Y})$, has been covered by Chart 86, and in Chart 87 is the means for evaluating the other line-constant factor, sinh $Vl$.

**Procedure.** Two separate operations are used in the chart, one for evaluating the magnitude and the other for the vector angle, and the former will be taken up first. A straight line is established through the scale points corresponding to the previously determined values of *Surge Impedance—Magnitude,* and *Sinh Vl—Magnitude,* on the scales so designated. Then, at the intersection of this line with the *Line Constant B—Magnitude* scale, that factor is read. This is a basic factor for the line and should be noted for any further studies for varying operating conditions of the line. Then, for any loading condition, a straight line is established between the scale point just found and the scale point corresponding to the known line-current numerical value. At the intersection of this line with the $I_2B$ scale may then be read the value of the second term of the voltage equation, as to magnitude.

It will be noted that the scale ranges for these various magnitudes are limited to a


LOAD POWER FACTOR & CURRENT PHASE ANGLE
LAGGING
LEADING
LOAD CURRENT-AMPS.
SINH V1-MAGNITUDE
SINH V1 PHASE ANGLE
$I_0$ B-MAGNITUDE
$I_0$ B PHASE ANGLE
LINE CONSTANT B-MAGNITUDE
LINE CONSTANT B PHASE ANGLE
SURGE IMPEDANCE-MAGNITUDE
SURGE IMPEDANCE PHASE ANGLE

single logarithmic range, but it is a simple matter to adjust for values beyond the actual scalings. The operation is direct multiplication, and if any of the factors has a value outside the scale range, it should be multiplied by such power of 10 as will give a value within those ranges, later multiplying the result by the product of the reciprocals of those multipliers.

Thus, if the actual value of surge-impedance magnitude fell in the range between 1 and 10 (the scaling being 10 to 100), the actual value would be multiplied by 10.0. Then, if the numerical value of sinh $Vl$ did fall within scale limits, the observed value of *Line Constant B* would have to be *divided* by 10.0. The same procedure may be followed with respect to the value of current. For instance, with values of *Surge Impedance* between 1 and 10, of *Sinh Vl* between 0.1 and 1.0, and of current between 10 and 100, the multipliers necessary to bring these values within the scalings would be 10.0, 10.0, and 0.1, respectively. Then the observed result would have to be multiplied by the product of the reciprocals of those multipliers, $0.1 \times 0.1 \times 10.0$, or 0.1.

The other phase of the use of the chart is in establishing the phase angles of line constant $B$ and of the vectorial product of this factor and that of load current. The first step is to fix a straight line through the scale points corresponding to the previously determined values for *Surge Impedance—Phase Angle* and *Sinh Vl—Phase Angle,* on the scales so designated. Then at the scale point of the intersection of this line with the *Line Constant B—Phase Angle* scale, that value is noted, to be used for any later studies with varying load conditions. For such studies, another straight line is fixed by this point and the scale point corresponding to the known value for *Load Power Factor,* or *Current—Phase Angle,* and at the intersection of this line with the *$I_2B$—Phase Angle* scale is read the vector angle for the second term of the basic equation.

To illustrate the use of the chart, the same set of conditions is assumed as for the charts immediately preceding, and with the load current assumed as 150 amperes at 80 per cent power factor, lagging, the angle being $-36° \ 52.18'$. Previous charts gave the value of $A$, or cosh $Vl$, as $0.96\underline{/1° \ 0'}$, $U$ as $420\underline{/-13° \ 50'}$, and sinh $Vl$ as $0.32\underline{/76.3°}$. Delivery voltage is taken as 150,000.

As stated previously, the evaluation of the first part of the voltage equation is without benefit of charts, being a simple multiplication of load voltage and term $A$. In this case, the product has a magnitude, from the above figures, of 144,000, and its vector angle, being that of factor $A$, is $1° \ 0'$, and this term is then fixed by these results as $144{,}000\underline{/1° \ 0'}$.

Proceeding to the second term of the equation, a straight line is established through the scale points for 42.0 (using a multiplier of 0.1) on the *Surge Impedance—Magnitude* scale and 3.2 (using a multiplier of 10.0) on the *Sinh Vl—Magnitude* scale. It so happens that in this case the effects of the multipliers are canceled, and at the intersection of this line with the *Line Constant B—Magnitude* scale is read the true value of this quantity as 135. This, then, is a basic constant factor for the line and may be used for any loading conditions. Taking up the single load condition assumed, another straight line is then fixed by this scale point and the scale point for 1.5 (using a multiplier of 0.01) on the *Line Current—Amperes* scale. At the intersection of this line with the *$I_2B$—Magnitude* scale is read 204, and to compensate for the use of the factors above

described, this must be multiplied by the product of their reciprocals, or 100, so the true value is 20,400, as the magnitude of the vector quantity.

Determining vector angles, a straight line is established through the scale points corresponding to $-13° 50'$ on the *Surge Impedance—Phase Angle* scale and 76.3° on the *Sinh Vl—Phase Angle* scale, and at the intersection of this line with the *Line Constant B—Phase Angle* scale, the value is read as 63°. This, again, is a basic line constant, to be used in computations for any loading condition. Again the assumed load is taken, and a straight line is fixed by the scale point just found and the scale point for 0.80 on the *Power Factor* scale or $-36° 52.18'$ on the *Current—Phase Angle* scale. At the intersection of this line with the *$I_2B$—Phase Angle* scale, the vector angle is then read as $+26° 0'$.

The vector quantities are then completely defined, the value of the line constant $B$ being $135.0\underline{/63° 0'}$, and, for the particular load condition, $I_2B$ being $20{,}400\underline{/+26° 0'}$. (Computation gives the figures as being $136.43\underline{/62° 55.35'}$, and $20{,}465\underline{/+26° 3.17'}$, respectively.)

It might be noted that while the magnitude scalings have been limited on the chart, in the interests of greater accuracy, it is believed that the scalings for vector angles cover the entire practical field, and no adjustment should be necessary. However, should the value of sinh $Vl$ be over 90°, it may be brought within the scale limits by subtracting from the true value any multiple of 90°, later adding the same angle to the final result, either line constant $B$ or $I_2B$.

By means of this chart the two terms of the voltage equation may be evaluated, but the addition of those terms, while possible by computation, may be simplified by the use of Chart 92 following. In the discussion accompanying that chart, the same set of conditions will be used by way of illustration as was used above, so, for reference, the equation for that particular set of conditions is set down here:

$$E_1 = 144{,}000\underline{/1° 0'} + 20{,}400\ \underline{/26° 0'}$$

The later chart provides a means for resolving these terms into in-phase and quadrature components, after which they may be added. Then, by means of the same chart, the total may be retransformed into a vector quantity to give a clearer picture of the final result.

# 90. Current-characteristic Factors (I)

**Method for the determination** of the first of the two terms in the solution of the equation for the generator or feed-end current, when the delivery-end conditions are known. Specifically, it provides for the evaluation of the first term on the right-hand sides of Eqs. (2), (4), and (6), in the discussion immediately preceding Chart 84. These equations are

$$I_1 = I_2 \cosh \sqrt{ZY}\, l + E_2\sqrt{Y/Z} \sinh \sqrt{ZY}\, l \qquad (2)$$

$$I_1 = I_2 \cosh Vl + E_2 W \sinh Vl \qquad (4)$$

$$I_1 = I_2 A + E_2 D \qquad (6)$$

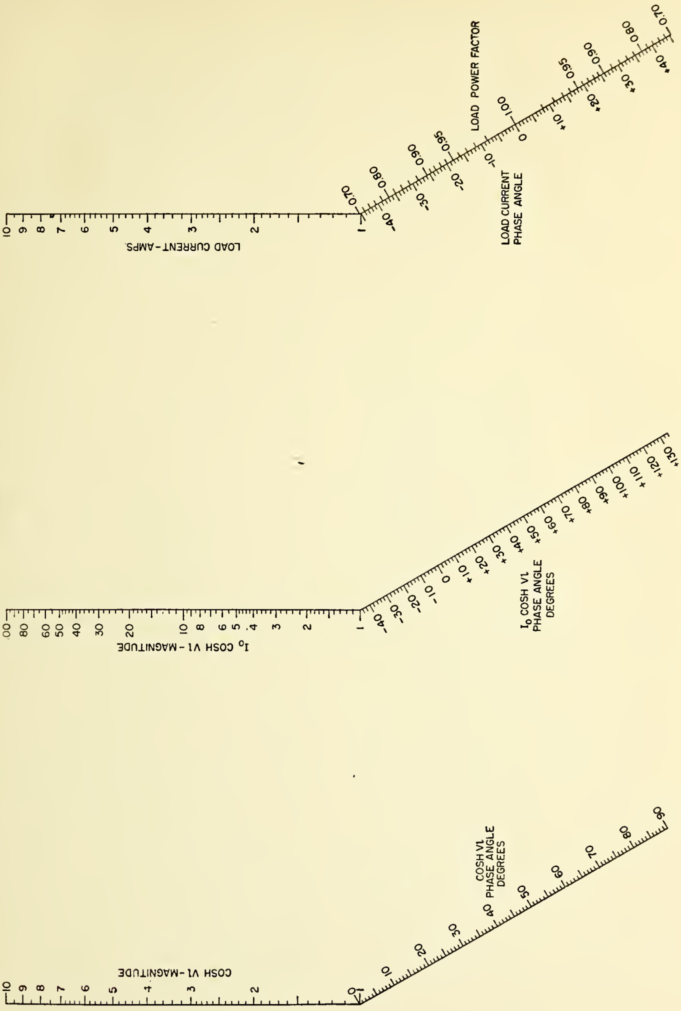
The term $I_1$ is the value to be determined for input-end current, and $I_2$ and $E_2$ are the current and voltage values at the delivery end, respectively. The terms in radical signs are functions of impedance and admittance, evaluated by means of earlier charts. They and their functions have been further simplified by substitution of terms $V$ and $W$. The first term, as it appears in Eq. (6), represented as $I_2A$, substitutes the factor $A$ for cosh $\sqrt{ZY}\, l$, of Eq. (2) or for cosh $Vl$ of Eq. (4). All quantities are vectors, having magnitude and phase angle, the basis of angular measurement being taken in assuming delivery voltage as having vector direction of zero angle.

**Procedure.** This chart is used to perform the vector multiplication of the factors $I_2$ and $A$. This procedure requires two separate operations, one to determine the magnitude of the product vector, and the other to determine the phase angle.

Taking first the determination of the magnitude of the vector quantity wanted, a straight line is established through the scale points corresponding to the known values for *Cosh Vl—Magnitude* (= cosh $\sqrt{ZY}\, l = A$) and *Load Current—Amperes,* and, at the intersection of this line with the $I_2$ *Cosh Vl* scale, the value is read for the magnitude of the vector quantity. Then, another straight line is fixed through the scale points for *Cosh Vl—Phase Angle* and for either *Load Power Factor* or *Load Current—Phase Angle,* on the scales so designated. At the intersection of this line with the $I_2$ *Cosh Vl—Phase Angle* scale, the vectorial angle of this quantity is read. So, then, this first term of the current equation is evaluated.

But to illustrate the use of the chart, numerical bases are brought in. The same set of conditions is assumed as in the earlier charts of this group. So, the load current, $I_2$, is taken as 150 amperes, at 80 per cent power factor (phase angle being $-36° \; 52.18'$), and cosh $Vl$ being 0.96 as to magnitude and having a phase angle of $1° \; 0'$. Both magnitudes fall outside the primary scale ranges, so a multiplier of 0.01 is applied to the *Load Current* factor and one of 10.0 to the *Cosh Vl* factor.

First, a line through the scale points for 1.50 on the *Load Current—Amperes* scale and for 9.6 on the *Cosh Vl—Magnitude* scale is found to intersect the $I_2$ *Cosh Vl* scale at a point indicating the magnitude of the vector as 14.3. But multipliers were used to bring the original factors within the scale limits, so the observed figure must


LOAD CURRENT-AMPS.
LOAD POWER FACTOR
LOAD CURRENT PHASE ANGLE
$I_0$ COSH VL - MAGNITUDE
$I_0$ COSH VL PHASE ANGLE DEGREES
COSH VL-MAGNITUDE
COSH VL PHASE ANGLE DEGREES

be multiplied by the product of the reciprocals of those multipliers. Thus, the figure 14.3 must be multiplied by 10.0, $(= 100 \times 0.1)$ to obtain the true magnitude, which becomes 143.0.

For the second stage, a straight line is fixed by the scale points for 1° on the *Cosh Vl—Phase Angle* scale and for 80 per cent on the *Load Power Factor* scale or $-36° 52.18'$ on the *Load Current—Phase Angle* scale. The result is read as 35.5°. Thus, the value of this first term is, by chart, $143.0\underline{/-35° 30'}$, where computation gives $142.77\underline{/-35° 52'}$.

# 91. Current-characteristic Factors (II)

**Method for the determination** of the second of the two terms in the solution of the equation for the generator or feed-end current of a transmission line, when the delivery-end conditions are known. Specifically, it provides for the evaluation of the second term on the right-hand sides of Eqs. (2), (4), and (6), in the discussion immediately preceding Chart 84. These equations are

$$I_1 = I_2 \cosh \sqrt{ZY}\, l + E_2\sqrt{Y/Z} \sinh \sqrt{ZY}\, l \tag{2}$$

$$I_1 = I_2 \cosh Vl + E_2W \sinh Vl \tag{4}$$

$$I_1 = I_2A + E_2D \tag{6}$$

The term $I_1$ is the value to be determined for input-end current, and $I_2$ and $E_2$ are the current and voltage values at the delivery end, respectively. The terms in radical signs are functions of impedance and admittance, evaluated by means of earlier charts. They and their functions have been further simplified by substitution of terms $V$ and $W$. The second term on the right-hand side of Eq. (6), represented as $E_2D$, substitutes the factor $D$ for $W \sinh Vl$ of Eq. (4), or $\sqrt{Y/Z} \sinh \sqrt{ZY\,l}$ of Eq. (2). The factor $D$, explained so, is a line constant and obtains regardless of load conditions, but it is combined with the voltage term in the interest of saving space in this collection of charts. All quantities concerned are vectorial, having both magnitude and phase angle, the basis of angular measurement being taken in assuming delivery voltage as having a vector angular direction of zero angle.

**Procedure.** This present chart is used to perform the vector multiplication of the factors, first to determine the line constant $D$, and then to determine the product $E_2D$ for any particular load condition. Each of these operations is twofold, first determining the magnitude, and second, the phase angle of the product.

The evaluation of the line constant $D$, because it will apply for any load condition, is first undertaken, both as to magnitude and angle. The first step, then, is to establish a straight line through the scale points corresponding to the known values for *Sinh Vl—Magnitude* and *Surge Admittance—Magnitude* (= $W$), on the scales so designated. Then, at the intersection of this line with the *Line Constant D* scale is read the magnitude of the vector quantity involved.

Next, another straight line is fixed by the scale points corresponding to the known values for *Sinh Vl—Phase Angle* and *Surge Admittance—Phase Angle* on their respective scales, and at the intersection of this line with the *Line Constant D—Phase Angle* scale is read the vector angle. Since the load-voltage vector angle has been assumed as the zero angle, the angle just found becomes the vector angle of the product of the line constant and the load voltage.

Having evaluated the line constant $D$, the evaluation of the second term of the equation under consideration requires, actually, but one more operation for each load-voltage condition. This operation consists in establishing a straight line

through the scale points for the previously determined values of *Line Constant D—Magnitude* and *Load Voltage,* on the scales so designated. Then, at the intersection of this line with the *$E_2D$—Magnitude* scale, the numerical value of the vector quantity is read. The vector angle of this quantity, as stated before, remains the same as the vector angle of line constant $D$. Thus, the second term of the current equation is completely evaluated as a vector quantity.

To illustrate the use of the chart, the same set of conditions is assumed as in the earlier charts of this group. $E_2$, the delivery voltage, remains at 150,000, with a vector angle of 0°. The term $W$, or $\sqrt{Y/Z}$, has been determined by Chart 88 as having magnitude of 0.00238 and vectorial angle 13° 45′. By means of Chart 87, sinh $Vl$ has been found to have a magnitude of 0.32 with a vector angle of 76.3°.

So, a straight line is established through the scale points for 3.2 (using a multiplier of 10.0 to bring the true value within the scale limits) on the *Sinh Vl—Magnitude* scale and 0.00238 on the *Surge Admittance—Magnitude* scale. Then, at the point of intersection of this line with the *Line Constant D* scale, a value for that constant is read as 0.0076. But since a multiplier of 10.0 was used to bring one of the factors within the scale limits, it is necessary to multiply the scale value by the reciprocal of 10, or 0.1, making the actual value 0.00076 (computed figure being 0.0007616).

Evaluating the phase or vector angle of this constant, a second straight line is fixed by the scale point for 13° 45′ on the *Surge Admittance—Phase Angle* scale and 76.3° on the *Sinh Vl—Phase Angle* scale. This line is found to intersect the *Line Constant D—Phase Angle* and *$E_2D$—Phase Angle* scale at a scale point indicating the vector angle for the line constant, as well as the final second term, as 90.6° (computed value being 90° 03′).

Thus, the line constant $D$ is now evaluated as $0.00076\underline{/90.6^\circ}$, and this value is taken for the determination of the second term in the equation for any load conditions. But, with the assumption that the load voltage is 150,000, another straight line is fixed through the scale point found above for the factor $D$, as 0.0076 (still using a multiplier of 10.0) and the scale point for 15,000 (again using a multiplier). This line is found to intersect the $E_2D$ scale at a point indicating the magnitude of 83.0.

The multipliers cancel out in this case, so this is the true value of the magnitude of this vector quantity. So the value of the second term of the equation is now fixed at $83\underline{/90.6^\circ}$, where the computed value is $85.80\underline{/90^\circ\ 3'}$.

The current equation is now resolved, upon the basis of previous computation, as

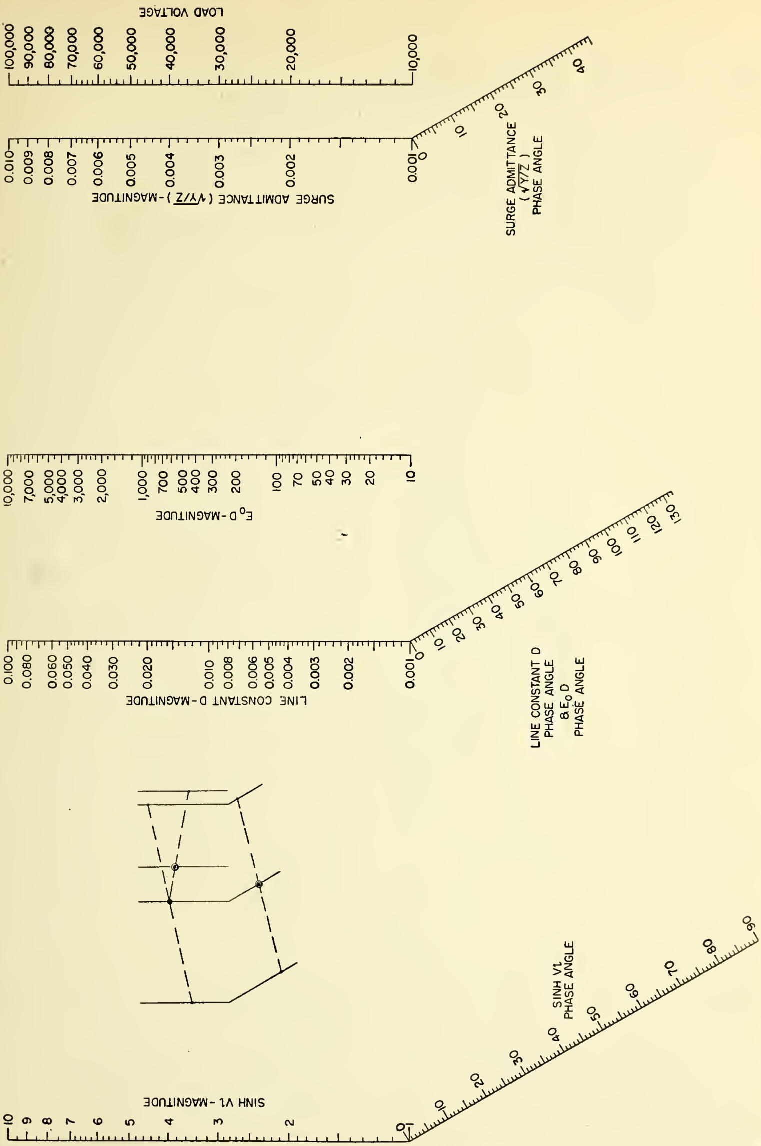
$$I_1 = 143.0\underline{/-35^\circ\ 30'} + 83.0\underline{/90.6^\circ}$$

by chart, and

$$I_1 = 142.77\underline{/-35^\circ\ 52'} + 85.8\underline{/90^\circ\ 3'}$$

by computation.

The addition of these vector quantities is resolved in the chart following, which also furnishes a means for the resolution of the voltage equation and so provides the completion of the solution of the two transmission-line equations.


LOAD VOLTAGE
100,000
90,000
80,000
70,000
60,000
50,000
40,000
30,000
20,000
10,000
SURGE ADMITTANCE ($\sqrt{Y/Z}$) - MAGNITUDE
0.010
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
SURGE ADMITTANCE
($\sqrt{Y/Z}$)
PHASE ANGLE
$E_0D$ - MAGNITUDE
10,000
7,000
5,000
4,000
3,000
2,000
1,000
700
500
400
300
200
100
70
50
40
30
20
10
LINE CONSTANT D - MAGNITUDE
0.100
0.080
0.060
0.050
0.040
0.030
0.020
0.010
0.008
0.006
0.005
0.004
0.003
0.002
0.001
LINE CONSTANT D
PHASE ANGLE
& $E_0D$
PHASE ANGLE
SINH VL - MAGNITUDE
10
9
8
7
6
5
4
3
2
1
SINH VL
PHASE ANGLE

# 92. Vector Equivalents

**Method for the final resolution** of the voltage- and current-characteristic equations of transmission lines. Earlier charts of the group have evaluated, as vector quantities, the two terms in each of the two equations, but to combine the terms, it is necessary to resolve the vector quantities into their respective rectangular-coordinate equivalents and, after addition, to retransform the results into vector quantities, to determine the quantities as to actual magnitude and phase-angle relation to the assumed basic phase angle.

In the discussion immediately preceding Chart 84, the two equations were given and then reduced to simplified forms, wherein the factors remaining constant in any line were described as line constants. These, for a given line, will apply for any loading condition, and, once they are determined for the line, solution of the equations for varying load conditions is reduced to the use of but a few of the charts of the series.

The derivation of the simplified equations has been described, as mentioned, and there is no need to repeat it here. But the final form is set down here:

$$E_1 = E_2A + I_2B \qquad (5)$$

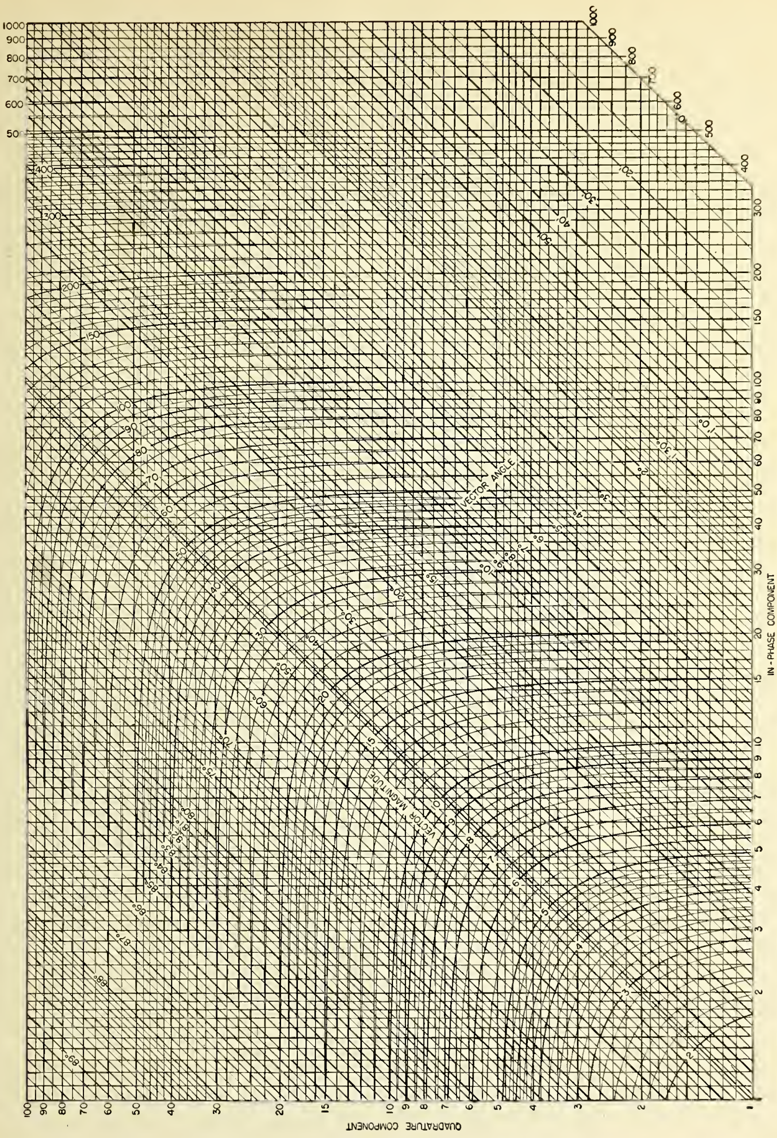
$$I_1 = I_2A + E_2D \qquad (6)$$

$E_1$ and $I_1$ are generator or feed-end voltage and current, respectively; $E_2$ and $I_2$ are load-end or delivery-end voltage and current, respectively; and $A$, $B$, and $D$ are line constants, evaluated by means of earlier charts of this group. All quantities are vectorial, having magnitude and phase angle with respect to a fixed base, $E_2$ being assumed as having zero angle.

Since each of the factors entering into each term of the equations is vectorial, the various products are also vector quantities. Means for determining the products of the various factors are also found in earlier charts for determining the vectorial values of those products. But the addition of the two terms in each of the equations requires a transformation into rectangular coordinates, after which the addition is accomplished by adding the in-phase components of the two terms (each equation) and adding the quadrature components, to obtain an in-phase and a quadrature component for the *sum*. This rectangular-coordinate expression for the result must then be retransformed to vector form to obtain a true picture of the result.

Obviously, the chart could not maintain any reasonable degree of accuracy and still cover the entire range of variables. Accordingly, a set of ranges was selected which is felt to offer reasonable accuracy, but which requires the use of multipliers or other adjustments to cover the entire field of variation. These, however, will be taken up as the individual steps in the use of the chart are discussed.

**Procedure.** To use the chart to transform a vector quantity into its rectangular-coordinate components, a point is located on the chart at the intersection (interpolating as necessary) of the curve (hyperbolas symmetrical about the sloping line noted as 45°) and the sloping line corresponding to


IN-PHASE COMPONENT
QUADRATURE COMPONENT
VECTOR ANGLE
VECTOR MAGNITUDE

*Vector Magnitude* and *Vector Phase Angle*, respectively. Then, the horizontal line through this point is a measure of the quadrature component and the vertical line through it is a measure of the in-phase component, and the values of those components are read at the left and bottom edges of the chart, respectively, interpolating, again, as may be required.

If the vector magnitude is beyond the scale limits, it should be multiplied by a factor such as will bring it within the scale limits, later multiplying the scale values of the quadrature and in-phase components by the reciprocal of that factor. If the vector angle is outside the scale limits, a more complicated system of adjustment is required.

Where the vector angle is between 90° and 180°, the supplement of the angle (180° minus the true angle) is used, in which case the quadrature component is read directly, but the in-phase component must be multiplied by $-1$. For angles between 180° and 270°, the angle used in the chart will be 180° less than the true angle, with the observed values of in-phase and quadrature components both being multiplied by $-1$. In cases of angles between 270° and 360°, the angle to be used in the chart will be the difference between the true angle and 360°, the in-phase component being as read, whereas the quadrature component must be multiplied by $-1$.

In some cases, where the angle to be used on the chart is near 90°, the complement of the angle (90° minus the angle) is used, in which event the quadrature component is read as being evaluated by the vertical lines, and the in-phase component is read as being fixed by the horizontal lines, the reverse of normal procedure. In some cases, adjustments may be required with both magnitude and angle to bring them within the scale limits, but each adjustment is made independently of the other.

Having determined the rectangular components of the two terms of either of the equations, the in-phase components of the two terms are added to obtain the in-phase component of their sum, and the quadrature components are added to obtain the quadrature component of their sum. Then, by reversing the earlier process, the vectorial value of the sum is obtained.

This is done by locating on the chart the intersection of the horizontal line corresponding to the quadrature component and the vertical line corresponding to the in-phase component, interpolating as necessary. Then, the magnitude of the vector of the desired figure is read from the curves, and the vector angle is read from the sloping lines, again interpolating as may be required. Of course, in this operation as well as the earlier ones, adjustments may be necessary to bring the quantities within the scale limits.

Thus, the solution of the characteristic equations for voltage and current has been accomplished, the results being in the form of vector quantities, the angles being with respect to delivery voltage as zero angle. Obviously, the difference between the two vector angles is the phase angle at the feed or generator end of the line.

By way of illustrating use of the chart, the same set of conditions is assumed as was used in earlier charts of this series. Thus, from Chart 89 (see its accompanying discussion), the equation for the feed-end voltage was determined as

$$E_1 = 144{,}000\underline{/1^\circ\,0'} + 20{,}400\underline{/26^\circ\,0'}$$

The values of both angles are within the scale limits, but it is necessary to multiply both magnitudes by 0.001 to bring them

within scale limitations, so the magnitudes are 144.0 and 20.4, respectively.

Interpolating between curves for a value of 144.0 for magnitude on the sloping line corresponding to 1° 0′, a point is then located on the chart. Then, by reading the scale values on horizontal and vertical lines (interpolating), the quadrature component is read as 2.50 and the in-phase component as 144.00 for the first term. Then, for the second term, a point is found where (interpolating) the curve value for 20.4 intersects the sloping line for 26° 0′. In this case, the horizontal and vertical lines indicate 8.85 and 18.20 as the quadrature and in-phase components, respectively, of the second term. Simple addition then gives the final result as having a quadrature component of 11.35 and an in-phase component of 162.20.

Again referring to the chart, the intersection is found for the horizontal line corresponding to 11.35 and the vertical line of 162.20, interpolating as required. This is found to be at a point on the sloping line for 4° 0′, and since no adjustment had been necessary with respect to angle, this is the true value. Interpolating between curves indicates a value of 162.25 as the magnitude of the vector, but since the original terms were multiplied by 0.001 to bring them within scale limits, the figure just obtained must be multiplied by 1,000. So, the voltage is now fixed as 162,250$\underline{/4^\circ\ 0'}$.

Turning now to the current equation, in the text accompanying Chart 91, the illustrative case showed the value of current as

$$I_1 = 143.0\underline{/-35^\circ\ 30'} + 83.0\underline{/90.6^\circ}$$

The angles for both terms fall outside the scale limits, so adjustment is necessary. As to the first term, the angle used on the chart will be the numerical value of the true angle, but it must be remembered that the quadrature component will be minus. For the second term, the angle being close to 90°, a chart angle of 0.6°, or 0° 36′ will be used, remembering that the quadrature component will be plus and the in-phase component will be minus and that the former will be evaluated by the vertical lines and the in-phase component by the horizontal.

Proceeding as before, the first term is found to have a quadrature component of −81.00 and an in-phase component of 113.50. Similarly, the second term is found to have a quadrature component of 83.00 and an in-phase component of −0.87. Simple additions evaluate the quadrature component of the final figure as 2.00 and the in-phase component as 112.63.

Again reference is made to the chart, and the intersection is found for the horizontal line for 2.00 and the vertical line for 112.63, interpolating as required. This point, in reference to the sloping lines, shows the vector angle as 1° 2′ and, in reference to the curves, shows the magnitude as 112.65.

This completes the determination of the generator or feed-end conditions for the assumed case. The voltage is 162,250$\underline{/4^\circ\ 0'}$, and the current is 112.65$\underline{/1^\circ\ 2'}$. The feed-end phase angle is then $\overline{2^\circ\ 58'}$. Actual computation gives the voltage as 161,130$\underline{/4^\circ\ 23.1'}$ and current as 114.29$\underline{/1^\circ\ 4'}$, the generator phase angle being $\overline{3^\circ\ 19.1'}$.

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